Deviations Due to the Discrete Modeling of the Structures

Binsar H. Hariandja, Ika Bali*

Department of Civil Engineering, President University, Cikarang, Indonesia

Received 10 February 2023; received in revised form 03 March 2023; accepted 10 March 2023

Abstract

Real existing structures are generally complicated in geometry that might make them unsuitable or difficult for the analyses. Several assumptions or simplifications are usually made by the practitioners to make them simpler and may be analyzed in ease. The assumptions and/or simplifications might make the analyses much easier; however, they might create some deviations from the true behavior of the structures, which are in fact, are not known precisely to the analysts. The paper discusses some examples for simplifications in the analysis or the structural systems. The examples include the types of the simplifications, and the deviations that caused by them. The types of simplifications are in the case of material behavior, the geometry of the system, an in the case of connectivity of the structural components. The results signify the importance of wisdom in the setting of the simplifications and the assumptions to make the analyses easier but with dependable results.

Keywords: discrete modeling, finite element, nonlinearity of structure

1. Introduction

The structural systems are the products of several engineering disciplines such as civil engineering, architect, electrical engineering and so on. As the products of several disciplines, the systems may become complicated. For civil engineering discipline for example, some efforts are needed to identify whether an item may be treated as structural or non – structural. More sophisticated architectural shape is cast in a simpler shape, and so on.

To narrow the scope of this paper, some simplifications that usually adopted in the modeling are presented. The aspects that will be referred to are, the modeling in the material behavior, the simplification of the shapes of the components, and in the connectivity of the components.

2. Modeling in Structural Analyses

As pointed out before, some simplifications and assumptions are usually made prior to the analysis procedure of a structure, to have a discrete model as a representative of the real structure in the analysis. The more crude the assumptions, the simpler the model that may be used, but the larger deviations that may result. In the following chapter, some assumptions that usually adopted in the analyses, are discussed. The aspects are in the material aspects, the shape of the components or the structures, and the connectivity of the structural components.

The needs for the simplifications of the real structural systems come from some sources. The most primary one, is that in old times, a power and useful tool for computations had not been yet invented. The tremendous effort in the computation seemed to be minimized by the use of simpler model as the representation of the real structures. The invention of

* Corresponding author. E-mail address: ika.bali@president.ac.id
Tel.: +62(0)21 89109763

computational tools such as slide rule and calculator, help the practitioners to use more complicated models.

The invention of computer in early 1940s and the use of matrix formulation in the analyses started in 1960s [1], is considered as a revolution in sciences, especially in engineering disciplines. In civil engineering, the use of computer and matrix formulation are combined in finite element method as a powerful numerical procedure.

3. Sources of Deviations

There are so many aspects that maybe the reasons for structural simplifications, but herein, some aspects will be discussed. They are the modeling in the material behavior, the modeling in the behavior of the components, and the modeling in connectivity among components.

3.1 The modeling in the material and structural behavior

The materials generally possess their respective physical and mechanical behavior. While still developing material engineering to understand better the real behavior of any old or new materials, there are so many attempts are adopted to simplify the properties of the material, so it can be used in a simpler step to go on simpler analysis.

The practice most often used is to assume that a material as an elastic one; and therefore probably, with elastic behavior of overall structure. By doing this kind of assumption, several merits may be enjoyed. The most one is that the elastic structure follow the principle of superposition. This superposition principle states that the total response of a structural system is mere an algebraic accumulation of individual responses to individual loading cases. Moreover, the total response does not depend on the sequence of the loadings. These two aspects very much reduce the computational effort in the analyses. This aspect attracts the practitioners to apply the linear elasticity assumption, even though the model behavior becomes quite far from the real behavior of the material.

3.2 The modeling in the behavior of the components

This aspect in fact relates to the availability of the computational tools. In old days, when the computational tools have not invented yet, the practitioners usually embarked to iterative computations such as Cross, Takabeya or Kani methods. The application of such iterative methods tends to use simple mode.

For example, consider the Cross method that widely applied by practitioners, in old days, particularly when sophisticate computational tools such as computers, were not exist yet. The method uses simple model, that only bending deformation that was accounted for, while axial deformation was ignored. This might give relatively accurate results for a structure with slender members, but for the structure with stocky or sturdy member, in which axial stiffness is relatively small relatively to bending stiffness, might give erroneous result.

3.3 The modeling in the connectivity among components

The capacity of a structure comes from the strength of individual components and the strength of the connections that assemble the structure. The connections may be construed as hinges, perfectly rigid or semi rigid. The connections have the function to transfer the internal forces among the adjacent components, until the forces are transferred to the ground via slabs, beams and columns or (shear) walls.

It may happen that a connection may be assumed to carry certain types of internal forces, but actually the connection behaves differently from the assumption or expectation. For example, a prestressed beam is considered as free from the column when the beam is prestressed. But in fact, the columns act to restrain the beam to achieve its elastic shortening for the prestressing force to work effectively. Another example is the simplification procedure that a prestressing beam is firmly supported by another crossing beam. But in fact, the crossing beam will experience uplift force in the connection between beams, due to the camber of the prestressed beam. The practitioners should not forget to inspect this case. These two cases are further discussed in the next chapter.

4. Several Examples

To illuminate the goals of the paper, several examples are described below. First, the snap through in a shallow plane truss problem. Secondly, the application of the Cross iterative analysis that neglecting the axial deformation. Third, a problem in which a prestressed concrete beam with reinforced concrete crossing beams. Lastly, a prestressed beam stressed while its ends connected to relatively rigid columns.

4.1 Snap through problem

The design of a relatively shallow plane truss, shown in Fig. 1, is based on an assumption that the structure behaves linearly elastic, and the practitioner come up with a relative slender component in a relatively shallow plane truss. For this case, as opposed to *infinite displacement*, *finite displacement* approach may be used. The finite strain for axial element is given by [2]

$$\varepsilon_{xx} = u_x + \frac{1}{2}(u_x^2);\ \ u_x = du/dx \tag{1}$$

in which u is the axal displacement. So, the strain in finite displacement scheme is nonlinear in displacement. To approach the condition, linearization in incremental approach is assumed. The displacement at time t is approached by

$$u^{t+\Delta t} = u^t + u \tag{2}$$

so the Almansi strain [3] is

$$u_x^{t+\Delta t} = u_x^t + u_x \tag{3}$$

According to standard procedure, the displacements are interpolated by the standard axial element shape functions as follows [4-8]

$$u^t = \left(1 - \frac{x}{L}\right) u_1^t + \frac{x}{L} u_2^t;\ u = \left(1 - \frac{x}{L}\right) u_1 + \frac{x}{L} u_2 \tag{4}$$

so the strains are

$$\varepsilon_{xx}^t = -\frac{1}{L} u_1^t + \frac{1}{L} u_2^t;\ u = -\frac{1}{L} u_1 + \frac{1}{L} u_2 \tag{5}$$

with the corresponding the Second Piola – Kirchoff stress [3]

$$S_x^{t+\Delta t} = S_x^t + S_x = E u_x^{t+\Delta t} = E u_x^t + E u_x \tag{6}$$

After run with a nonlinear analysis program, the load – displacement is obtained as shown in Fig. 2. Even though the connection is placed above the support level, the force will render the connection down, and up to a stage of the loading, the connection will suddenly deflect downward, the case usually referred to as *snap through*. The blue curve is for the moderate plane truss, whereas the red curve relates to the shallow plane truss case. If the structure is designed as an elastic system, the shallow plane truss will not be able to achieve prescribed design load level.

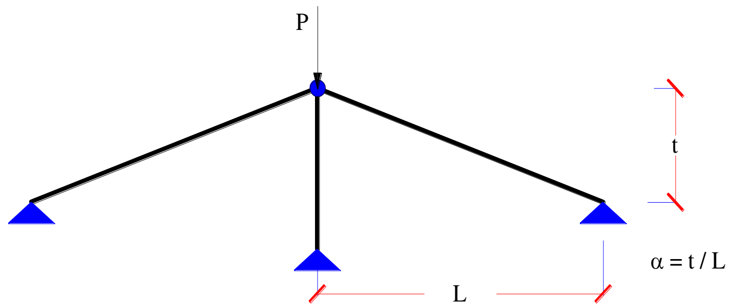

Fig. 1 Shallow plane truss structure

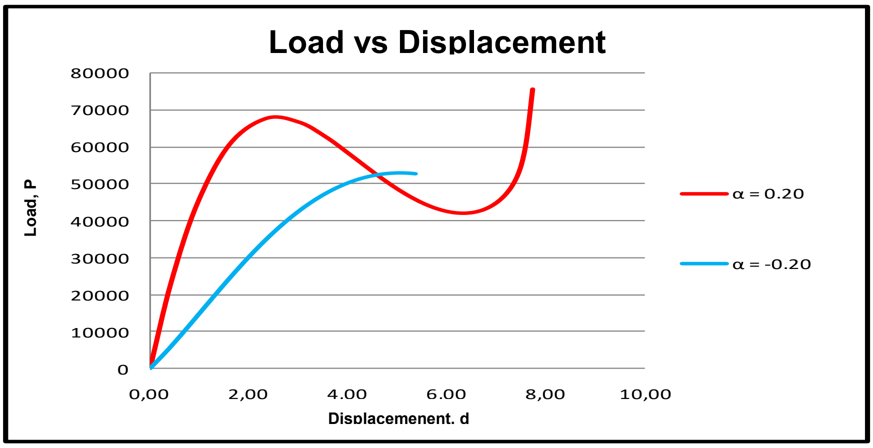

Fig. 2 Snap through in shallow plane truss

4.2 The application of cross method to plane frame

The method was invented by Hardy Cross, far before computational tools such as calculator or computer were invented. To make the analysis simpler, only bending deformation is considered, while the axial deformation is neglected. Moreover, the elongation of the member is also neglected. These assumptions are identical with the assumption that the axial stiffness is extremely large. In this case, after the solution of bending moments, the shear and even axial forces are computed simply by using bending solution. This is true for the shear, but actually is quite not true for axial force, as will be shown in the following discussion.

Now, consider simple plane frame with the dimensions of the columns $b \; x \; d = 0.30 \; m \; x \; 0.30 \; m$ and the beam $b \; x \; d = 0.30 \; m \; x \; 0.30 \; m$, the height of the frame is $H = 4.0 \; m$ as shown in Fig. 1, Several cases are considered, i.e., the inclination of the column $\emptyset = 45^0, 60^0, 75^0,$ and 90^0. The length of the column and the span of the beam are the same, i.e., L. The horizontal displacements and the moments of the column tip due to a horizontal concentrated load $P = 1000 \; kN$ at node 2, are observed, and tabulated in Table 1. The analyses are carried out by two schemes. The first is done by the application of the Cross method, and the second by the use of finite element method (FEM).

Table 1 The horizontal displacement and moment of column tip

$\emptyset$		45^0	60^0	75^0	90^0
Cross	$\Delta_h (cm)$	0.703	2.677	2.652	2.836
	M (kN-m)	1778.9	1342.6	1252.7	857.0
FEM	Δ_h (cm)	0.947	1.331	1.892	2.836
	M (kN-m)	525.0	645.0	757.0	857.0

In the first case, the axial stiffness of columns and also the beam do not contribute to the response of the structure to the point load. For the case $\emptyset = 45^0$, the horizontal displacement of column tip is $\Delta_h = 0.703\ cm,$ and the moment in the beam end and column tip is $M = 1778.9\ kN - m.$ In the second case, the axial stiffness of columns and also the beam contribute to the response of the structure to the point load. For the case $\emptyset = 45^0$, the horizontal displacement of column tip is $\Delta_h = 0.947\ cm,$ and the moment in the beam ends and column tips is $M = 525.0\ kN - m.$ For the case $\emptyset = 90^0$, both cases give quite similar results. The horizontal displacement of column tip is $\Delta_h = 2.836\ cm,$ and the moment in the beam ends and column tips is $M = 855.0\ kN - m.$

Nowadays, the presence of the computer as a powerful computational tool permits the practitioners to more precisely model a structure in the analysis. Even though in the old days, such powerful computational tools did not exist, the application of the Cross method was still acceptable, since the columns in the buildings usually are placed vertically.

4.3 Prestressed concrete beam crossing with reinforced concrete beams

A floor plan with dimension $6.0\ m\ x\ 12.0\ m$ is supported by a prestressed concrete beam in the $10.0\ m$ direction and a reinforced concrete beam in the $6.0\ m$ direction. The size of the prestressed concrete beam is $b\ x\ d = 0.30\ m\ x\ 0.60\ m$, and the size of reinforced concrete beam is $b\ x\ d = 0.25\ m\ x\ 0.40\ m$. Both beams are made of concrete with the quality $f_c' = 40.0\ MPa.$ The prestressed concrete beam is designed with a prestressing force $F = 1{,}600\ kN$, by using a tendon with an eccentricity $e = 0.24\ m$ at mid span section. This prestressing force gives moment distribution

$$M(x) = \frac{4x(L-x)}{L^2} F\ e \tag{7}$$

and an uplift uniformly distributed force

$$q_F = -\frac{8\ F\ e}{L^2} = -\frac{8\ x\ 1600\ x\ 0.24}{(12.0)^2} = -\ 21.33\ kN/m \tag{8}$$

After the prestressing concrete beam is placed, the reinforced concrete poured but the slab floor is not yet poured on, the forces at the prestressing and reinforced concrete beam are

$$\begin{aligned} q_{pc} &= 0.3\ x\ 0.6\ x\ 24.0 - 21.33 = \ -17.01\ kN/m \\ q_{rc} &= 0.25\ x\ 0.4\ x\ 24.0 \qquad\quad = \ +\ 2.40\ kN/m \end{aligned} \tag{9}$$

The beam crossing system is then analyzed by considering consistency of displacement at crossing point. The vertical displacement of the prestressed concrete beam at the crossing point is

$$v_{pc} = -\frac{5q_{pc}L_{pc}^4}{384EI_{pc}} - \frac{RL_{pc}^3}{48EL_{pc}} \tag{10}$$

In which R is the vertical reaction between prestressed concrete beam and reinforced concrete beam.

The vertical displacement of the reinforced concrete beam at point C is

$$v_{rc} = -\frac{5q_{rc}L_{rc}^4}{384EI_{rc}} + \frac{RL_{rc}^3}{48EL_{rc}} \tag{11}$$

The consistency of displacement dictates that the two vertical displacements in Eqs. (10) and (11) are the same, so

$$R = \frac{\frac{5q_{rc}L_{rc}^4}{384EI_{rc}} - \frac{5q_{pc}L_{pc}^4}{384EI_{pc}}}{\frac{5q_{rc}L_{rc}^4}{384EI_{rc}} + \frac{5q_{pc}L_{pc}^4}{384EI_{pc}}} \tag{12}$$

which gives

$$R = \frac{\frac{5q_{rc}L_{rc}^4}{384EI_{rc}} - \frac{5q_{pc}L_{pc}^4}{384EI_{pc}}}{\frac{5q_{rc}L_{rc}^4}{384EI_{rc}} + \frac{5q_{pc}L_{pc}^4}{384EI_{pc}}} \tag{13}$$

This force creates secondary negative moment at section C in the reinforced concrete beam with the amount

$$M_{rc} = -123.4\ kN - m \tag{14}$$

Since that section is considered as simple support in the analysis and is not provided with the tensile reinforcement, the beam would crack at that particular section.

4.4 Prestressed concrete beam with columns at both ends

In normal practices, a prestressed concrete component is usually cast as an isolated unit, precast, prestressed and afterwards, positioned at proper place and then assembled it to the other components; in this case, columns. However, the case will be entirely different if the hardened beam is poured monolithically with the columns and then stressed. This the case that is discussed in the next explanation.

A floor with large dimensions is supported by the columns spaced $16.0\ m\ (ctc)$. So, large span beam is used between two adjacent columns. Unfortunately, the beam is stressed after its end are poured monolithically with columns. The size of the columns is $b\ x\ d = 0.60\ m\ x\ 0.60\ m$ and the height is $H = 3.50\ m,$ while the prestressing concrete beam dimension is $b\ x\ d = 0.60\ m\ x\ 0.80\ m.$ The quality off the concrete for the two components is $f_c' = 4.0\ MPa.$ The prestressing beam is compressed with a prestressing force $F = 4250\ kN.$

Now, the restrain force due to the axial stiffness of the prestressing beam and the lateral stiffness of the column is

$$R = \left(2\frac{12EI_c}{H^3} + \frac{EA_b}{L}\right)\Delta \tag{15}$$

So, the force taken by the column is

$$R_c = \frac{2\frac{12EI_c}{H^3}}{2\frac{12EI_c}{H^3}+\frac{EA_b}{L}}F = 712.80\ kN \tag{16}$$

which is about 16.77% of the total prestressing force. This reduction should be accounted for as an addition to the prestressing loss, that already summing up to about 15 ~ 20% loss before this additional loss of prestress.

5. Conclusions

From all of the discussions presented above, several conclusions are drawn as follows.

- Several examples that signify the deviations due to the discrete modeling of true existing structural systems, are presented.
- The examples consist of an example of that the system is assumed to be linearly elastic, an example of not accounting axial deformation in plane frame system, the floor with the assumption that crossing beams are just connected by hinge connection, and the prestressing beam is stressed while already monolithic with columns.

For continuing future studies or practices, some aspects are proposed as follows.

- Due to the presence of sophisticate computational tools such as computer, the more delicate modeling of the structural systems with less assumptions, may be dealt with in ease.
- The practices of constructing his or her own computer programming in the analysis of the structures, are good practices, since our dependability to other computers program may be minimized.

Acknowledgments

The authors express their appreciation to the Head of the Civil Engineering Department of President University by supporting the subscription to the Fortran Application. The authors also extend their sincere appreciation for the excellent typing is carried out by Mrs. Okke Tumbuan.

References

[1] J. M. Argyris, and S. Kelsey, Energy Theorems and Structural Analysis, London: Butterworth, 1960.
[2] B. Hariandja, Mekanika Bahan dan Pengantar Kepada Teori Elastisitas, Jakarta: Penerbit Universitas Pancasila, 2010.
[3] L. E. Malvern, Introduction to the Mechanics of a Continuous Medium, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1969.
[4] R. D. Cook, D. S. Malcus, and M. E. Plesha, Concept and Application of Finite Element, 3rd ed., New York: John Wiley & Sons, Inc., 1989.
[5] R. H. Gallagher, Finite Element Analysis, Fundamentals, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1975.
[6] B. Hariandja, Metoda Elemen Hingga: Teori dan Konsep Dasar, Jakarta: Penerbit Universitas Pancasila, 2015.
[7] B. Hariandja, Mekanika Rekayasa IV: Metoda Matrix Dalam Analisis Struktur, Bandung: Penerbit Institut Teknologi Bandung, 2015.
[8] O. C. Zienkiewicz, and R.L. Taylor, The Finite Element Method, New York: McGraw-Hill Company, 1991.