RPLiDAR-Based Mapping in Development of a Health Service Assisting Robot in COVID-19 Pandemic

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***Abstract*—Due to the Covid-19 pandemic, robots with various functions in the health sector are growing, one of which is autonomous robots capable of supporting logistics, drug and food delivery, and monitoring the environment. In this study, a map-based indoor navigation system was developed on health service assisting robot based on LiDAR and Robot Operating System (ROS) platform. The robot with the SLAM GMapping algorithm was succeeded to perform mapping with an accuracy of 95.983%. Then the robot with an automatic navigation system using the AMCL Particle Filter algorithm for localization was succeeded to perform localization with an error of 1.3377 on the x-axis and 1.2109 on the y-axis and 45,72570 for orientation on the z-axis. And succeeded to do trajectory tracking using the move_base package for path planning with an error of 1.6477 on the x-axis and 0.4498 on the y-axis when moving towards the target coordinates.**

Keywords—autonomous navigation, LiDAR, mapping, localization, trajectory tracking

I. Introduction

Historically, robots have provided many benefits to humans in a variety of fields, given their ability to perform tasks with precision, perform industrial operations efficiently, interact in hostile environments, and perform highly complex jobs [1]. Robots in the medical field were firstborn in 1985 with the function of performing brain biopsies through robotic arm technology with the help of computerized tomography (CT) imaging with a preprogrammed system for its movement [2].

As technology evolves, artificial intelligence (AI) with computer vision and data analytics have transformed the world of robotics in the health field, such as the development of robots, which can help perform sanitation work and perform medical examinations automatically, detecting pneumonia caused by viruses [3]. Even with AI, it can expand the use of technology to many other areas of healthcare, such as robots for surgery [4], radiology [5], all the way to the world of medical education [6].

Not only that, but there is also the use of the Internet of Things, such as drones for surveillance to ensure quarantine and the use of masks, big data to predict the spread of viruses, then virtual reality, holography, cloud computing, autonomous robots (autonomous robots), 3D scanning, 3D printing, and biosensors [3].

Now robots are not only used in operating rooms, but also in clinical settings to support health workers and improve patient care [2]. During the Covid-19 pandemic, hospitals began utilizing robots for a broader range of tasks to help reduce the overload of health workers, reduce the risk of transmission in the hospital environment, the availability of biomedical technology, and the sustainability of patient care [7]. This shows that work efficiency and risk reduction provided by robots in the field of health provide many benefits [8]-[10].

For example, robots that use ultraviolet (UV) light, evaporation techniques, and vacuum cleaners to ensure disinfection or sterilization to clear pathogens, which can help reduce the risk of transmission [8], [11]. Then the robot that able to send samples, food, medicine, medical supplies, to record the patient's condition automatically [12]-[14]. This was supported in a study involving 41 health professionals (nurses, doctors, biomedical engineers, and others) showing that during a pandemic 65.8% of doctors recommend the use of robots capable of supporting logistics tasks, drug, and food delivery, as well as monitoring the environment [7].

As technology evolves, robots will function more independently, ultimately performing certain tasks entirely on their own. As a result, doctors, nurses, and other health workers may focus on patient care [12]. Robots that can work independently are like autonomous robots and others.

Autonomous robots have been used in a variety of fields, from industry, hospitality, to health environments with light detection and ranging (LiDAR) sensors as their main components [15]-[27]. The application of LiDAR in robotic navigation systems has a huge impact in predicting environmental conditions in real-time because LiDAR has very high accuracy, particularly for short-range measurements in room mapping [28]-[30]. Robot Operating System (ROS) is used to perform mapping using LiDAR sensors, considering that in recent years the use of ROS has become increasingly widespread in the development of robots [31].

Autonomous robots using LiDAR sensors include mobile robots [17]-[22], omnidirectional mobile robots [23], [24], telepresence robots that can interact with patients using LCDs [16]. The robot successfully navigates, avoids obstacles, and travels automatically to a designated point. The error rate is about 1% to 6% [18], [20], [23]. The robot is equipped with infrared and can provide better results [25]. Not only that, LiDAR sensors can be combined with the camera and use neural networks to process the data. The system provides an accuracy of about 90% [26], [27].

II. Methodology

A. Environment

Mapping and testing of the robot navigation system were carried out in the hall of the Wisma Teladan housing complex, Bandung as shown in Fig. 1. The testing arena consisted of five rooms whose frames were made of wood and plastic to wrap them with dimensions of about 2 x 1 x 1 m. It aims to determine whether the robot can walk to the intended room automatically properly. The shape of the arena is important because this robot is designed as a health service assisting robot that will work in hospitals. The test arena is shown in Fig. 1 and the camera direction when Fig. 1 was captured is shown in Fig. 2, with (a) is the camera direction when Fig. 1 (a) was captured and forth. The physical form of the robot is shown in Fig. 3.

Fig. 1. Robot testing arena in three different direction

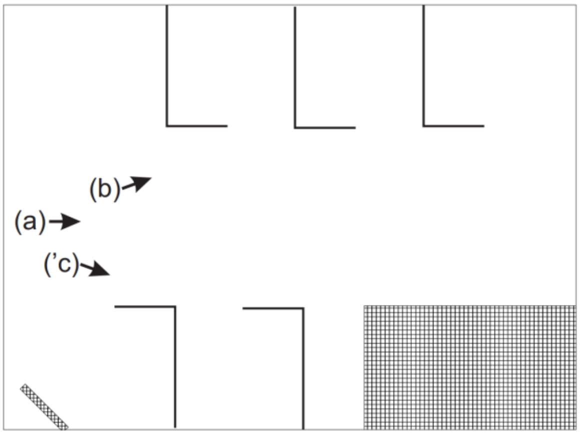


Fig. 2. Plan of the robot testing arena with camera direction when Fig. 1 was captured

Fig. 3. The physical form of health service assisting robot

B. Robot System

To navigate, the robot is equipped with a RPLiDAR A1M8 laser rangefinder sensor which is capable to perform a 360-degree scan within a distance of 12 m with a scanning frequency of up to 5.5 Hz. This robot is also equipped with Intel NUC that serves as the main controller, Arduino Mega 2560 as a secondary controller which controls the motor driver and receives the value of the number of wheel rotations from the rotary encoder. The robot system diagram is shown in Fig. 4.

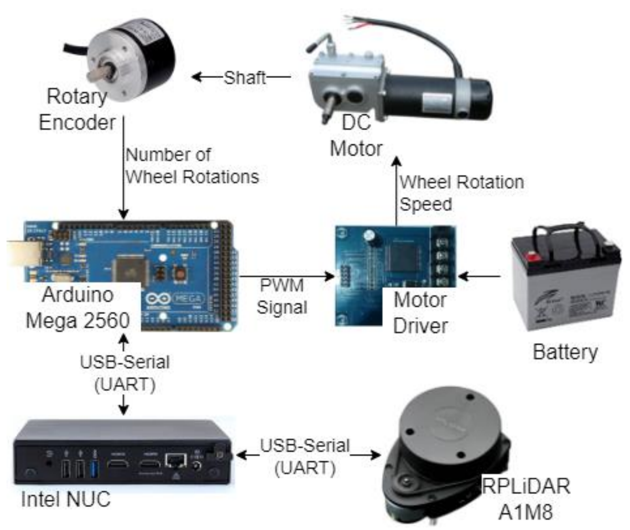


Fig. 4. Robot system diagram

Intel NUC installed the Ubuntu 20.04.3 (Focal Fossa) operating system to run ROS Noetic. Intel NUC as the main controller is in charge of mapping, localization, path planning, and trajectory tracking. When the robot performs trajectory tracking, the Intel NUC will provide a control signal to the Arduino Mega 2560 to drive the motor so that the robot can move along the path of the path planning result.

C. *Navigation System*

The navigation system based on ROS implemented on the robot includes mapping, localization, path planning, and trajectory tracking. ROS is a flexible Linux-based framework for robotics development. ROS contains a collection of tools, libraries, and conventions that can overcome the complexities of building a good robot [32].

To run autonomous navigation, a map from the mapping results is required first. The mapping is done using the SLAM GMapping algorithm. The robot moves using manual control via a teleoperated keyboard through the corridor and into the room with several speeds to get the best mapping results and determine the effect of robot speed on the accuracy of mapping by LiDAR.

The localization process is carried out to calculate the relative position of the robot. The relative position of the robot is obtained through the AMCL Particle Filter algorithm by processing wheel odometry data. Wheel Odometry calculates the position of the robot based on the rotation of the wheel obtained from the rotary encoder which produces data on the linear speed of the robot on the x and y axes, the angular speed of the robot on the z-axis, and the angle of orientation of the robot on the z-axis (θ or yaw) [33].

The mapping results are then recorded and displayed on RViz (visualization tool on ROS). The distance obtained using LiDAR is compared with the distance measured using a meter manually to determine the accuracy of the measurement and mapping results.

Once the map is acquired, the target coordinates (x, y) and the desired final orientation angle (θ or yaw) can be entered visually with RViz. Based on the initial position, target position, and environmental map, the robot will perform path planning based on the move_base package on the ROS which connects global planning and local planning.

Dynamic Window Approach (DWA) is an algorithm that is used for the planner. After the path planning result is obtained, the robot will perform trajectory tracking to move along the path resulting from the path planning until it reaches the target position. The navigation system connection diagram on the ROS is shown in Fig. 5 and the robot navigation system flowchart is shown in Fig. 6.

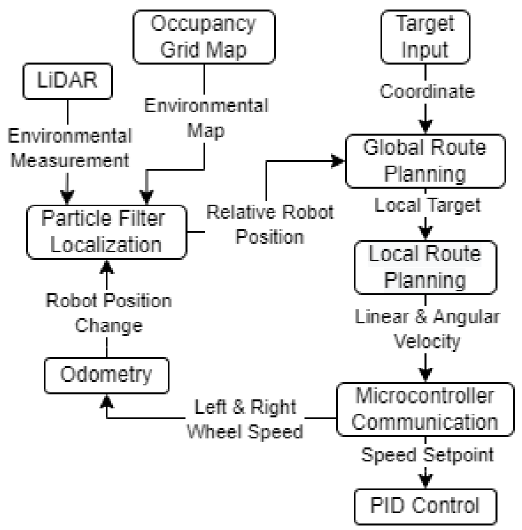


Fig. 5. Navigation system connection diagram on ROS

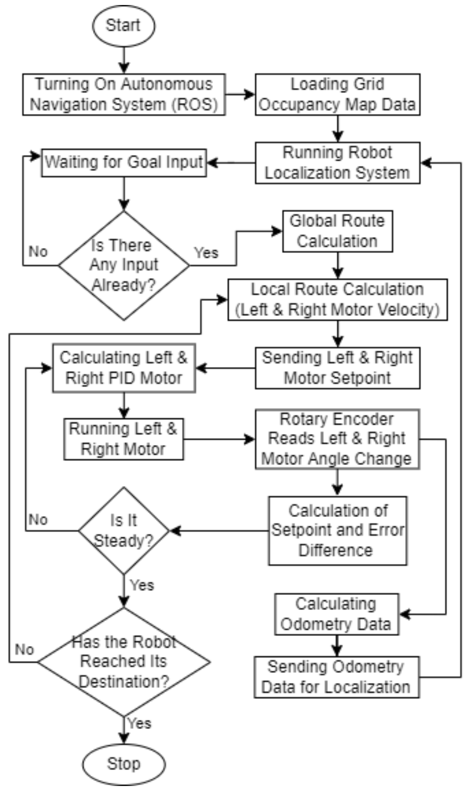
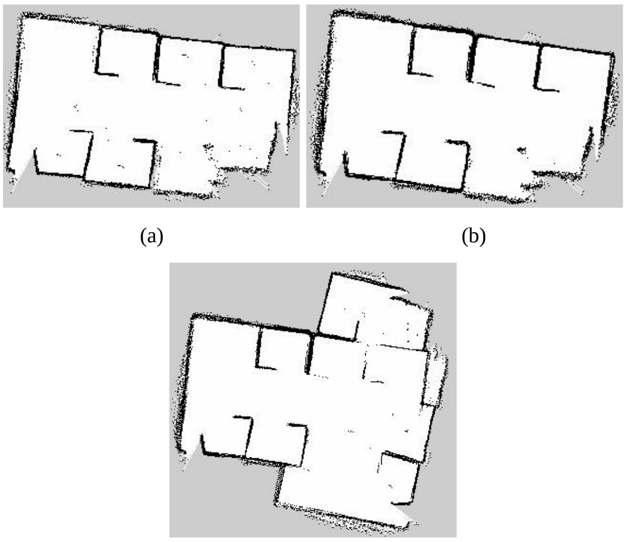


Fig. 6. Robot navigation system flowchart

III. Results and Analysis

A. *Mapping Testing*

Mapping tests were conducted three times at different speeds. The speed values of the robot during mapping testing are 0.2 m/s; 0.5 m/s; and 0.7 m/s. The selection of the speed value is done by trying to do a mapping from the smallest speed to the value where the mapping results become bad and not suitable as a navigation reference because this test aims to get the best mapping results. Then the best mapping results are used as a reference when the robot runs an automatic navigation system and the robot does not need to do the mapping continuously as long as the area used does not change. The results of the mapping test are shown in Fig. 7.



(a) (b) (c)

Fig. 7. Mapping results: a) 0.2 m/s; b) 0.5 m/s; c) 0.7 m/s

The results in Fig. 7 show that the higher speed of the robot when mapping, the less good the map results, as in the highest speed variation (0.7 m/s), the map of the mapping results stacked. This is because a scan matching error occurred during mapping because the robot was running too fast and the computer was not able to perform too fast computations so that it produces the wrong position and grid.

Then measurements are taken at several points on the map of the mapping result and compared to measurements manually in the testing area using the tape measure. Measurement of the mapping results is carried out through RViz by selecting the points to be compared. The measurement results using the tape measure are used as the actual value or reference in calculating the error value and the accuracy of the mapping results. The measurement points compared to determine the mapping accuracy are shown in Fig. 8 and the comparison of the measurement results is shown in Table I.

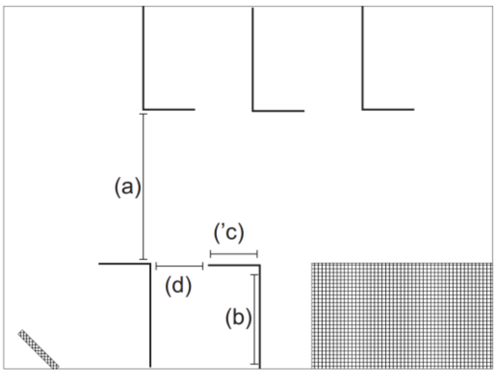


Fig. 8. Compared measurement points, marked by points (a), (b), (c), and (d).

TABLE I. COMPARISON OF MEASUREMENT RESULTS USING TAPE MEASURE AND LIDAR

0.2 m/s				
Measurement Points	**Measured using Tape Measure (cm)**	**Measured on RViz (cm)**	**Error (cm)**	**Accuracy (%)**
(a)	280	256.209	23.791	90.714
(b)	203	204.335	1.335	99.347
(c)	103	99.242	3.758	96.213
(d)	177.5	181.756	4.256	97.658
Average			8.285	95.983
0.5 m/s				
Measurement Points	**Measured using Tape Measure (cm)**	**Measured on RViz (cm)**	**Error (cm)**	**Accuracy (%)**
(a)	280	264.250	15.750	94.040
(b)	203	204.180	1.180	99.422
(c)	103	94.773	8.228	91.319
(d)	177.5	179.335	1.835	98.977
Average			6.748	95.939

Based on Table I, the comparison between measurements obtained through LiDAR and manual measurements using a tape measure shows that the highest accuracy is obtained when the speed is 0.2 m/s, followed by 0.5 m/s with a very small difference in accuracy. The speed value of 0.5 m/s can be used for mapping in the indoor area such as healthcare setup because it has high accuracy and the mapping process does not take too long compared to the speed of 0.2 m/s. For a map when the speed is 0.7 m/s, the measurement value is not compared because the mapping result is stacked.

B. *Localization Testing*

Localization testing is carried out when the robot moves from the starting point to the coordinates specified by autonomous navigation. The test was carried out twice with two different target coordinates. In the localization process, AMCL particle filter processes data obtained from wheel odometry based on the number of wheel rotations. The data on changes in the relative position of the robot during walking were recorded as shown in Fig. 9. The data was recorded in a ROS bag file which was then converted to CSV and compared between the robot's average relative position from wheel odometry and AMCL particle filter as shown in Table II.

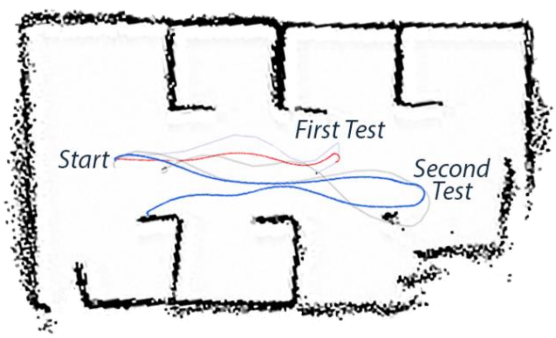


Fig. 9. Executed navigation trajectory

In Fig. 9 it can be seen that the trajectory data produced by wheel odometry (red and blue bold line) and AMCL particle filters (black thin line) are different. The difference is also shown in Table II where the average robot position value from wheel odometry and AMCL particle filter has an error of 1.3377 on the x-axis and 1.2109 on the y-axis and 45.7257^0 for orientation on the z-axis. This is because the wheel odometry data is sourced from the wheel, which is easy to make mistakes when doing trajectories, such as slipping, and others. The error is cumulative so that it affects the entire data.

AMCL particle filter successfully performs localization by estimating the position and orientation of the robot while moving and understanding the environment, which is proven in testing the robot successfully runs the automatic navigation system and advances to the specified target position.

TABLE II. COMPARISON OF LOCALIZATION RESULTS ON WHEEL ODOMETRY AND AMCL PARTICLE FILTER

Tracking Test	Wheel Odometry			AMCL Particle Filter			Error		
	X (m)	Y (m)	Orientation (degree)	X (m)	Y (m)	Orientation (degree)	X (m)	Y (m)	Orientation (degree)
First Test	1.3016	-0.7956	65.0778	3.9539	-2.4596	31.6356	2.6523	1.6640	33.4422
Second Test	1.0144	-3.3051	81.7097	0.9914	-2.5472	23.7005	0.0230	0.7579	58.0091
Average Error							1.3377	1.2109	45.7257

C. *Trajectory Tracking Testing*

To run the robot with automatic navigation, the coordinates of the target on the global planner must be determined first via RViz. Then the robot moves when it has successfully carried out path planning based on the move_base package planner and trajectory tracking testing is carried out. While the robot is moving towards the target, wheel odometry calculates the change in the robot's relative position from the rotation of the wheel. Then the data from the wheel odometry is processed on the AMCL particle filter which is then compared to see the difference in the target coordinates on the global planner and the final coordinates when the robot runs on the AMCL particle filter as shown in Table III.

TABLE III. COMPARISON OF TARGET AND FINAL COORDINATES ON THE ROBOT

Tracking Test	Target Coordinate		Final Coordinate		Error	
	X (m)	Y (m)	X (m)	Y (m)	X (m)	Y (m)
First Test	3.0355	-2.4626	4.0239	-2.5207	0.9884	0.0581
Second Test	1.4001	-2.2630	3.7070	-1.4215	2.3069	0.8416
Average Error					1.6477	0.4498

The comparison of the target coordinates on the global planner and the final coordinates on the AMCL particle filter is quite small, namely 1.6477 on the x-axis and 0.4498 on the y-axis. The difference in these values can be caused by the large tolerance value of the target position and an error when the robot has finished navigating.

In testing the robot successfully runs the automatic navigation system and moves to the specified target position without hitting an obstacle in the static environment so that it can be said that the robot has succeeded in trajectory tracking.

The robot also can be performed in dynamic environments because LiDAR with DWA planner has the ability to detect moving or sudden objects. When that happens, the planner will reroute to reach the destination.

IV. CONCLUSION

Health service assisting robot with a ROS-based automated navigation system equipped with LiDAR sensors successfully mapping and running autonomous navigation system. The highest mapping accuracy reached 95.983% for the robot's speed variation of 0.2 m/s. However, the best mapping speed value for healthcare setup is at 0.5 m/s with an accuracy value of 95.939% because it has an accuracy value that is almost the same as that produced at a speed of 0.2 m / s but can do mapping faster.

For autonomous navigation systems, localization testing errors are 1.3377 on the x-axis and 1.2109 on the y-axis, and 45.72570 for orientation on the z-axis. For trajectory tracking testing there are quite minor errors, namely 1.6477 on the x-axis and 0.4498 on the y axis. However, in testing the robot successfully moved towards the specified target coordinates without hitting the obstacle.

Further research can be done by looking for other influences on the mapping process produced using RPLiDAR A1M8, such as the slope of the area. The effect of speed on the mapping process can be different if using other types of LiDAR so that research can be done to see the influence of speed and others on the mapping process. The design of the robot framework can be improved to be more effective and efficient for running the autonomous navigation system.

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