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Modeling bitcoin price by using Euler-Maruyama method

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Abstract— In this study, we use Euler-Maruyama method to simulate Bitcoin prices dynamics. We investigate a year-long movement of Bitcoin prices. Daily closing prices were collected over a period starting from May 27th, 2023 and ending on May 27th, 2024. This data provides a comprehensive picture of how Bitcoin behaved on a daily basis throughout that specific year. The Euler-Maruyama method is used as numerical method for solving stochastic differential equations (SDEs). The method involves discretizing the SDE into an iterative process to obtain a simulated price trajectory. The drift term was estimated by the average daily return of Bitcoin prices over the study period. The volatility term was estimated by the standard deviation of daily Bitcoin returns. A Monte Carlo simulation was performed to generate a range of possible price trajectories using the Euler-Maruyama method. The result shows that the Euler-Maruyama method was able to capture the short-term trend of Bitcoin prices effectively.

Keywords— *bitcoin, stochastic differential equations, Euler-Maruyama method, Monte Carlo simulation*

I. INTRODUCTION

In the financial world, cryptocurrencies have become a disruptive force that is catching the interest of both individuals and organizations. They are digital assets that provide a decentralized substitute for conventional, centrally-controlled currencies. They are defined as digital assets that use cryptography for safe transactions and creation control [1]. Since its launch in 2009, Bitcoin, the creation of the anonymous Satoshi Nakamoto [2], has grown to be synonymous with cryptocurrencies, despite being relatively new in many nations. The innovative use of blockchain technology by Bitcoin, which creates a distributed ledger that documents every transaction made on the network [3], has opened the door for a radical change in financial systems. Many other cryptocurrencies (altcoins) have been developed as a result of the popularity of Bitcoin. Young adults in particular are drawn to these coins because of their potential as investments and as a means of learning new technical skills [4]. The recent surge in the prices of Bitcoin and altcoins like RNDR, Dogecoin, and LINK highlights their allure [5]. However, this rapid growth comes with a significant caveat: volatility. Cryptocurrency prices can fluctuate dramatically in short periods, indicating a high-risk investment landscape [6].

Interest in creating precise forecasting models for Bitcoin prices has increased due to the cryptocurrency market's explosive growth [7]. Because of the complicated dynamics and inherent volatility of Bitcoin prices, this work is difficult. Numerous statistical and machine learning techniques, each with advantages and disadvantages, have been put forth to tackle this problem. Time series forecasting has historically made use of statistical techniques like Autoregressive Integrated Moving Average (ARIMA) [8]. These strategies are based on statistical assumptions about the distribution of the data and the correlations between past and future values. For capturing linear relationships and short-term trends, ARIMA models are effective. Statistical methods, however, are not very good at handling non-stationary data or capturing complex nonlinear relationships. The volatility and nonlinearity of bitcoin prices may make it difficult for conventional statistical models to capture their behavior. Because machine learning techniques, especially neural networks [9], can infer complicated patterns from data without making explicit assumptions, they have become extremely useful for time series forecasting. In order to overcome this constraint, recurrent neural networks (RNNs) [10] include feedback connections, which enable them to store data about previous inputs and use it for prediction purposes.

The mechanisms of the price movements of bitcoin are complex and include both predicted tendencies and ostensibly random variations. In order to represent these dynamics, this work puts forth a remarkably straightforward model called Brownian motion. It is unexpected to learn that Brownian motion, a mathematical process that resembles random walks, can reflect stochastic processes (by a random diffusion term) as well as predictable trends (via a drift term). We solve this model with the Euler-Maruyama method in an attempt to show that the behavior of the Bitcoin price can be well captured by even a simple method.

II. RESEARCH METHODOLOGY

This study explores the application of stochastic models, particularly those incorporating Brownian motion, to predict bitcoin prices. A stochastic differential equation (SDE) model forms the foundation of this approach, where random disturbances are added to deterministic equations to model stock price dynamics. The Euler-Maruyama method approximates solutions to these SDEs by discretizing time and iterating through small steps, utilizing statistical properties of observed data. This method is augmented with Monte Carlo simulations, which generate numerous potential price trajectories through repeated random sampling, providing insights into the range of possible outcomes. By simulating Bitcoin prices using the Euler-Maruyama method and comparing these simulations to actual market data, the study evaluates the model's accuracy in capturing both average trends and inherent randomness.

1. Stochastic model with Brownian motion model

Regarding a model, in this case, a stochastic differential equation can be obtained by adding a random disturbance term to a deterministic differential equation [11]. In general, the differential equation in Stochastic Models has the form:

$$dX(t)= \mu(t, X(t))\, dt + \sigma(t,X(t))\, dB(t) \tag{1}$$

where

$X(t)$: Stock price at time t.
μ : Drift, representing the expected return of the stock
σ : Volatility or measure of fluctuation of stock price.
B : The differential dB(t) of Brownian motion B(t), also known as white noise.

The drift and volatility can be assumed to be constant values, the Brownian motion can be read as

$$dX = \mu X dt + \sigma X dB \tag{2}$$

where μ and σ are obtained as fixed values from observed data.

Equation (1) can also be written in the form of the following integral equation, meaning that the integral form is the analytical solution of the model. The integral form is written as follows

$$X(t) = X(0) + \int_0^t \mu(s, X(s))ds + \int_0^t \sigma(s, X(s))dB(s) \tag{3}$$

where X(0) is the initial value of X at the point $t = 0$. If we observe the second term in the integral equation, it is a regular integral that is deterministic in nature. We can use simple Riemann integral to calculate the second term. However, the integral in the third term contains the stochastic process $W(t)$ meaning that the integral is not regular integral. The integral is known as Ito integral [12]. The integral form can be used to obtain solution of dynamical model in order to have an analytical expression [13].

2. Euler-Maruyama Method

The Euler-Maruyama method approximates the solution $X(t)$ at discrete time steps. Suppose we divide the time interval $[0,T]$ into N small steps of size $\Delta t=T/N$. Then, the method updates the process X at each step using the following iteration:

$$X_{t+\Delta t} - X_t = \mu X_t \Delta t + \sigma X_t \Delta B \tag{4}$$

where ΔB is the increment of Wiener process, is a continuous-time stochastic process starting at zero with independent and normally distributed increments, having stationary increments and almost surely continuous paths [12], on $[t, t + \Delta t]$. This value can be approximated as $\Delta B=(\Delta t)^{1/2} Z_t$, where Z_t is variable taken from standardized normal distribution. The expression in equation (4) is very similar to finite difference method as in [14].

3. **Monte Carlo simulation**

The Monte Carlo method is a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results, particularly useful for solving problems that are deterministic in principle but difficult or impossible to solve analytically. Named after the Monte Carlo Casino in Monaco, this method reflects the element of chance associated with random sampling. [14] It involves generating random samples from a probability distribution to simulate the behavior of a complex system or process. By analyzing the outcomes of these random samples, one can estimate various numerical quantities, such as means, variances, and probabilities. The accuracy of the Monte Carlo method improves with an increasing number of samples, as repeating the process many times helps converge towards the true value of the quantity being estimated.

4. **Return value of Stock Price**

Stock price represents the current market value of a company's shares, reflecting investors' expectations about the company's future earnings and growth potential. Tracking the stock price is crucial for investors, as it provides a snapshot of the company's financial health. However, simply observing the stock price is not sufficient for making informed investment decisions. Instead, investors often focus on returns, which measure the percentage change in the stock price over a specific period. This approach allows for a more meaningful comparison of performance across different stocks and timeframes, regardless of their initial prices.

To calculate the return of a stock, we take difference of stock price at $t + \Delta t$ and t relative to ratio with price at t, *i.e.*

$$\text{Return} = (X(t + \Delta t) - X(t))/X(t) \tag{5}$$

This formula yields the relative return, which expresses the gain or loss as a proportion of the initial investment. For instance, if a stock price increases from \$100 to \$110, the return is (110−100)/100=0.10 or 10%.

Relative return is a valuable metric because it standardizes the performance measure, making it easier to compare the efficiency of different investments. Unlike absolute price changes, relative returns account for the initial value of the investment, enabling investors to gauge how effectively their capital is being utilized. This comparative perspective is essential in a diversified portfolio, where investments span various sectors and asset classes. By focusing on relative returns, investors can better assess risk-adjusted performance and make more informed allocation decisions.

Bitcoin Price Data

Data for this analysis is obtained from Yahoo Finance. However, data can also be retrieved from Trading View, a reputable website known for providing reliable data similar Trading View can be used to view cryptocurrency price movements or cryptocurrency charts. Data can be selected within various timeframes such as minutes, days, weeks, months, and years.

In this study, the data used consists of daily Bitcoin price from 27 May 2023 to 27 May 2024. The data can be seen in Table 1. We focus on Close value of bitcoin price. The table also contains return value which calculated by using equation (5).

Table 1. Bitcoin Price (USD), Source: yahoo finance

Date	Open	High	Low	Close	Return
27/05/2023	26.720,18	26.888,88	26.621,14	26.868,35	-
28/05/2023	26.871,16	28.193,45	26.802,75	28.085,65	0,05
29/05/2023	28.075,59	28.432,04	27.563,88	27.745,88	-0,01
30/05/2023	27.745,12	28.044,76	27.588,50	27.702,35	0
31/05/2023	27.700,53	27.831,68	26.866,45	27.219,66	-0,02
01/06/2023	27.218,41	27.346,11	26.671,72	26.819,97	-0,01
02/06/2023	26.824,56	27.303,86	26.574,64	27.249,59	0,02

03/06/2023	27.252,32	27.317,05	26.958,00	27.075,13	-0,01
04/06/2023	27.075,12	27.407,02	26.968,22	27.119,07	0
05/06/2023	27.123,11	27.129,98	25.445,17	25.760,10	-0,05
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The bitcoin price over the observed time can be seen in Figure 1. The figure shows the daily Bitcoin price over the course of a year. Starting at around $30,000, the Bitcoin price remains relatively stable within the $30,000 to $40,000 range for the first 100 days, exhibiting some fluctuations. From Day 150 to Day 250, there is a significant upward trend, with the price sharply rising to over $50,000. This period is marked by notable volatility, including steep rises and corrections. Towards the end of the year, from Day 300 to Day 365, Bitcoin reaches a peak of around $70,000, with the price showing some volatility but generally staying within the $60,000 to $70,000 range. The plot highlights Bitcoin's high volatility, showing substantial price increases and corrections, reflecting the asset's sensitivity to market sentiment, news, and broader economic factors. Overall, the year saw a significant rise in Bitcoin's price, more than doubling from its initial value.

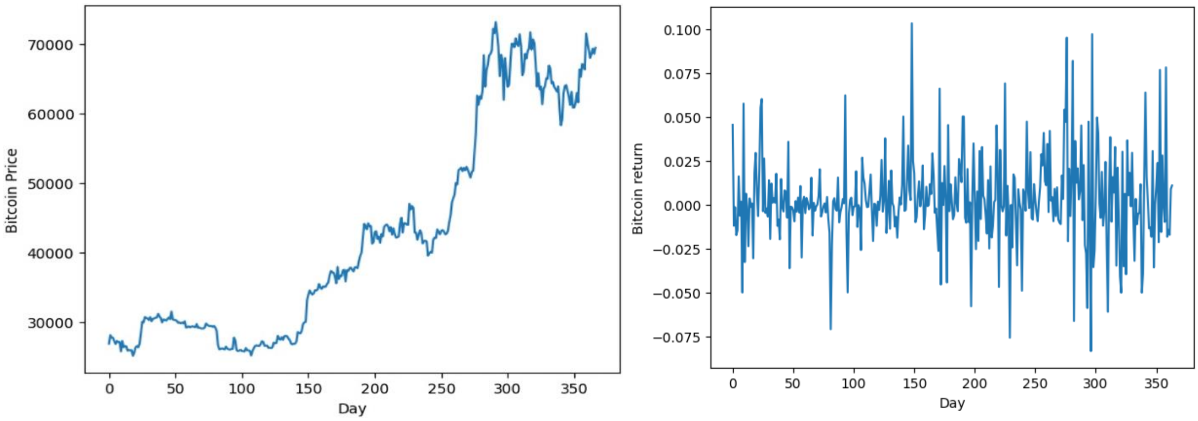

Figure 1. Daily bitcoin price over one year (left) and its return (right)

III. RESULTS AND DISCUSSION

The Euler-Maruyama method is a used for simulating the price trajectory of assets like Bitcoin (BTC) using stochastic differential equations (SDEs). To apply this method, we first initialize several parameters. We define the total time period T and set N as the length of the observed BTC closing prices array (`closeBTC`). The time step $\Delta t = 1/N$, and an t is created to represent discrete time interval (array).

Next, we calculate the statistical properties of the BTC returns. Specifically, we compute the mean and the standard deviation of the BTC returns. The mean is then scaled by N, and the standard deviation is scaled by the square root of N. These calculations are crucial for capturing the drift and volatility components of the BTC price movement in the SDE.

To simulate the stochastic part of the SDE, we generate a Brownian motion path. This involves setting a random seed for reproducibility and generating an array `dB` of increments for the Brownian motion, scaled by square root of Δt and random normal variables (with mean and standard deviation obtained from BTC returns). The cumulative sum of `dB` gives us the Brownian motion path `B`, which introduces the randomness into our BTC price simulation.

The simulation is done by the following steps:

1. Initialize parameters:

    returnBTC : array of return values of Bitcoin close price

    T : 1

N	: number of data
dt	: 1.0 / N
t	: array from dt to 1 + dt with step size dt

2. Calculate statistical properties:

μ_{BTC} = mean(returnBTC) $\cdot N$

σ_{BTC} = std(returnBTC) $\cdot$ sqrt(N)

3. Generate Brownian motion:

dB = sqrt(dt) $\cdot$ random normal array of size N
B = cumulative sum of dB

4. Euler-Maruyama approximation:

Set X = first element of closeBTC
for j in range(0, N):

$$X(j+1) = X(j) + \mu_{BTC} \cdot X(j) \cdot dt + \sigma_{BTC} \cdot X(j) \cdot dB(j)$$

The Euler-Maruyama method provides a numerical solution for simulating the BTC price trajectory by combining deterministic and stochastic components. This approach allows us to model the complex dynamics of BTC prices, capturing both their average behavior and the inherent randomness observed in real-world financial markets. The result from the algorithm can be seen in Figure 2.

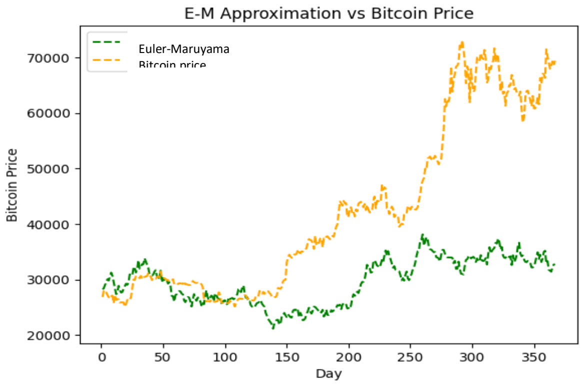

Figure 2. Comparison of Actual Bitcoin Prices with Euler-Maruyama Model Predictions

Figure 2 depicts the numerical results obtained from the Euler-Maruyama method. The predicted Bitcoin prices (Euler-Maruyama approximation) exhibit fluctuations due to the inherent randomness of financial markets. Notably, for the initial 100 time steps, the model closely aligns with the actual data, suggesting effective short-term trend capture. However, beyond this point, the model diverges significantly, attempting to catch up but still displaying substantial errors. This divergence may stem from model limitations or unaccounted market dynamics.

Monte Carlo Simulation on the Euler-Maruyama Method

The Monte Carlo simulation using the Euler-Maruyama method provides an alternative perspective on the dynamics of Bitcoin's price movement. Figure 3 shows 200 simulations of the algorithm, we can see that the true outcome is uncertain and can be influenced by various factors. However we can see that the actual data lie in the range of results obtained from all simulations. This shows that all of the simulation exhibit both lower and higher price alternatives. Meaning that we can see that the actual data falls within this range, which suggests that our simulation is capturing some of the key drivers of Bitcoin's price movement.

By taking the average of each time step for all simulations, we can generate a blue line that closely follows the trend of Bitcoin's price movement. This suggests that our simulation is fairly accurate in capturing the increase in price over time. This result is particularly good, as it indicates that our model is able to capture the general direction of price movements, even if it may not be able to predict specific highs and lows.

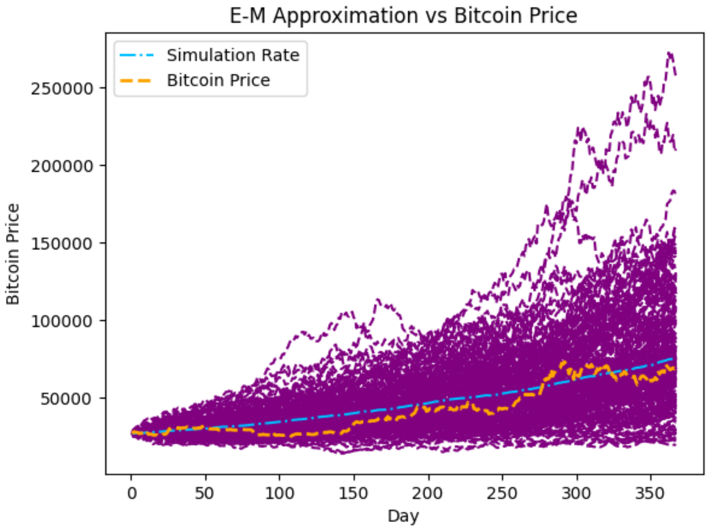

Figure 3 Monte Carlo Simulation's result from the Euler-Maruyama Method

IV. CONCLUSION

Stochastic differential equations were used to simulate the price trend of Bitcoin using the Euler-Maruyama approach. Initializing parameters, computing statistical characteristics of the Bitcoin returns, creating a Brownian motion path, and applying the Euler-Maruyama approximation were all part of the simulation. The model successfully captured the short-term pattern of Bitcoin's price movement, but it diverged beyond 100 time steps, according to the data.

Using a Monte Carlo simulation, consists of 200 numerical experiments, of the Euler-Maruyama method was run to provide multiple alternatives on the dynamics of the price movement of Bitcoin. The results from all the simulations fit the real data, suggesting that the model accurately predicted certain significant trends in the movement of the price of Bitcoin. Moreover, averaging every simulation of every time steps in the simulations produced outcomes that closely matched the trajectory of Bitcoin's price movement, indicating that the model did a reasonable job of capturing the price growth over time.

This study shows capability of the Euler-Maruyama method to simulate the price trajectory of bitcoin and identify its pattern. Despite the model's limitations which only rely on constant observed parameters, the results suggest that it can be an advantageous tool for understanding and predicting changes in the price of bitcoin. Further study could focus on improving the model by involving non-stationary parameters.

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