Journal of Actuarial, Finance and Risk Management (JAFRM)
Vol. 2(2), Desember, 2023.

Annual Premium Calculation on Whole Life Single Life Insurance Using Gompertz Mortality Assumption

Michelle Novia[1*], Fauziah Nur Fahirah Sudding[2]

[1,2,3] *Study Program of Actuarial Science, School of Business, President University, 17550, Indonesia*

[*] *Corresponding author: michelle.novia@student.president.ac.id*

Abstract— Premium calculation is one of the important aspects to insurance companies. Given the importance of premiums in insurance contracts for insurance companies, determining the price of the premium must also be appropriate. Careless determination of the premium price can cause the insurance company to fail to bear the risk that the company has. There are several ways to determine premium payments. In this research the premium calculation will be computed using Gompertz mortality assumptions which will be applied to the annual premium calculation of whole life single life insurance of man and woman. The benefit assumed, interest rate, Insurer age, Gompertz parameter and several actuarial notations such as life annuity-due and net single premium is needed in the premium calculation using Gompertz mortality assumptions. This research uses the data of Indonesian Mortality table (TMI IV) and the Linear Least Squares (LLS) method to find the Gompertz parameter which then be used to find the life annuity-due that will be needed to compute the premium calculation of Gompertz assumptions. Based on the calculation performed in this research, the value of the premium using Gompertz assumptions is influenced by parameters on the Gompertz assumptions, the interest rate used, and the Insured age.

Keywords— Premium Calculation; Gompertz Mortality Law; Whole life Insurance; Single Life

I. INTRODUCTION

Learning from the pandemic of Covid-19, which was declared a global pandemic by WHO on March 11, 2020 [1]. This incident had an impact on human health and activities throughout the world, including the financial issue of the world. This event shows that risks can occur unexpectedly, anytime, anywhere in the future. Adverse events cannot be stopped but losses can be minimized. One of the measures to minimize financial risks that can occur in the future is insurance. Life insurance is closely related to risk control in life. Life insurance itself is a safety program in the formation of passing on the economic risk of an insured person's death or life, and premium payments will stop at that time. Based on the number of insureds, life insurance is divided into two, single life (individual) insurance, where the number of insured is one insured and multiple life for more than one insured. Multiple life insurance is divided into joint life insurance and last-survivor life insurance. Based on the length of coverage, life insurance is divided into four types: Whole Life Insurance, Term Life Insurance, Endowment Insurance and Unit Link Life Insurance [2].

Given the importance of premiums in insurance contracts for insurance companies, determining the price of the premium must also be appropriate. Careless determination of the premium price can cause the insurance company to fail to bear the risk that the company has. Therefore, it is necessary to calculate the premium to get a premium price under the company's risk. According to research by [2], the amount of premium paid is influenced by costs, probability of death (mortality), interest rate, and the amount of compensation to be received. To determine premium payments, initial life annuity value which are influenced by the probability of survival and the probability of death is also required. Several methods can be used to calculate the probability of death and the probability of survive, one of which is the Gompertz assumption [3].

Gompertz law of mortality is one of the mortality laws that is used to compute actuarial values. Gompertz's Mortality Law was introduced by an English scientist Benjamin Gompertz in 1952 and is often used to find the value of the survival function of a population based on existing death data. To calculate the survival function, the Gompertz distribution assumes that the mortality rate increases exponentially with the age of an individual [3]. Moreover, the Gompertz model is also able to produce a great fit to life table data over some age ranges, especially from mean age to early old age [4]. The use of the Gompertz assumption in this research is also because this distribution is a fairly accurate distribution in describing the mortality rate of a population with only two parameters, compared to the Makeham and Weibull distribution which needs to use three parameters [5].

II. LITERATURE REVIEW

A. Survival and Distribution Function

Let X be a continuous random variable that states the *age at death*. The probability of newborn will die within x years is:

$$F_X(x) = Pr(X \leq x), \qquad x \geq 0 \tag{1}$$

This function is the distribution function of X. The probability of newborn will attain age x is expressed by the survival function as follows [3].

$$S_X(x) = 1 - F_X(x) = Pr(X > x), \qquad x \geq 0 \tag{2}$$

The relationship between distribution function and survival function is given by the following equation:

$$F_X(x) + S_X(x) = 1 \tag{3}$$

The probability of an individual aged x years dying in the next t years is expressed by the distribution function of $T(x)$ as follows [4].

$$\begin{aligned} F_{T(x)}(t) &= \Pr[T(x) \leq t] \\ &= \Pr[X - x \leq t | X > x] \\ &= \frac{\Pr[x < X \leq x + t]}{\Pr\,[X > x]} \\ &= \frac{S(x) - S(x+t)}{S(x)} \\ &= {}_tq_x \end{aligned} \tag{4}$$

The probability of an individual aged x years survive until the next t years is expressed by the survival function of $T(x)$ as follows:

$$\begin{aligned} S_{T(x)}(t) &= \Pr[T(x) > t] \\ &= \Pr[X - x > t | X > x] \\ &= \frac{\Pr[X > x + t]}{\Pr\,[X > x]} \\ &= \frac{S(x+t)}{S(x)} \\ &= {}_tp_x \end{aligned} \tag{5}$$

Based on (3), (4), (5), the following relationship can be obtained

$${}_tq_x + {}_tp_x = 1 \tag{6}$$

${}_tq_x = probability\ of\ death$

${}_tp_x = probability\ of\ survive$

Curtate future lifetime of x is a discrete random variable denoted by K_x which is the number of future years finished by x prior to death.

B. Force of Mortality

Mortality rate is a value that defines the probability that an individual will die within a short time near the future. Consider an individual with age x. The time interval that an individual will die is denoted by Δx. The definition of the mortality rate function using (4) is expressed as [3]:

$$\begin{aligned} \Pr[x < X \leq x + \Delta x | X > x] &= \frac{\Pr\,[x < X \leq x + \Delta x]}{\Pr\,[X > x]} \\ &= \frac{\Pr[X \leq x + \Delta x] - \Pr\,[X \leq x]}{1 - F_X(x)} \\ &= \frac{F_X(x + \Delta x) - F_X(x)}{1 - F_X(x)} \\ &\cong \frac{f_X(x)\Delta x}{1 - F_X(x)} \end{aligned} \tag{7}$$

From the equation (7), $F'_X(x) = f_X(x)$ is the probability density function of the continuous age-at-death random variable. The equation in (5) has a conditional probability density explanation. Denoted by μ_X is the value of the conditional probability density function of *X* at exact age *x* for each age *x*, given survival to that age [3].

$$\mu_x = \frac{f_X(x)}{1 - F_X(x)}, \quad \mu_x \geq 0$$
$$= \frac{-S'(x)}{S(x)} \tag{8}$$

From Equation (8), we can get [6]:

$$\mu_x = \frac{-S'(x)}{S(x)}$$
$$= -\ln\,[\mathrm{S}'(x)]$$
$$= -\ln\,[{}_xp_0] \tag{9}$$

After integrating both sides

$$\int_0^x \mu_y dy \quad = -\ln\,[{}_xp_0]$$
$$\exp\left(\int_0^x \mu_y dy\right) = \exp(-\ln[{}_xp_0])$$
$${}_xp_0 = \exp\left(-\int_0^x \mu_y dy\right) \tag{10}$$

Equation (10) can also be written in the following form:

$${}_tp_x = \exp\left(-\int_x^{x+t} \mu_y dy\right) \tag{11}$$

The equation (11) can be rewritten as (Bowers et al., 1997):

$${}_tp_x = \exp\left(-\int_0^t \mu_{x+s}\, ds\right) \tag{12}$$

C. Gompertz Law of Mortality

One of the mortality assumptions is Gompertz assumptions. The Gompertz distribution was introduced by an English scientist Benjamin Gompertz in 1952 and is often used to find the value of the survival function of a population based on existing death data. To calculate the survival function, the Gompertz distribution assumes that the mortality rate increases exponentially with the age of an individual. When the age of the Insured is $x + t$ years, the mortality rate function is expressed as follows [3]:

$$\mu_{x+t} = Bc^{x+t} \tag{13}$$

With B as the initial level of mortality and c is the rate at which mortality increases with age. B and c are parameters that are estimated based on mortality data. Based on equation (13) survival function and Gompertz distribution can be found [3].

with $B > 0, c > 1, x \geq 0$

When the age of the Insured is $x + t$ years, the mortality rate function is expressed as follows:

$$S(t) = \exp\left(-\int_0^t \mu_{x+s}\, ds\right)$$
$$= \exp\left(-\int_0^t Bc^{x+s}\, ds\right)$$
$$= \exp\left(-\frac{Bc^x}{lnc}(c^t - 1)\right) \tag{16}$$

with $B > 0, c > 1, x \geq 0$

D. Linear Least Squares (LLS)

Simple linear regression can be written as follows [7]:

$$E[Y_i|x_i] = \beta_0 + \beta_1 x_i \tag{17}$$

The expected value of Y given x, where x_i for $i = 1,2,\dots,n$ is a constant variable and Y_i for $i = 1,2,\dots,n$ is a random variable. The LLS method is a method that minimizes the *Sum of Squares Error* (SSE) that is the sum of the squares of the vertical distances between the actual observed variables, namely $y_i(x_i)$. Therefore, the LLS method will minimize the following SSE:

$$\begin{aligned} SSE &= \sum_{i=1}^{n} \left[y_i(x_i) - E[Y_i|x_i]\right]^2 \\ &= \sum_{i=1}^{n} [y_i(x_i) - (\beta_0 + \beta_1 x_i)]^2 \end{aligned} \tag{18}$$

The estimators for β_1 and β_0 are denoted by $\hat{\beta}_1$ and $\hat{\beta}_0$. The derivatives of SSE in equation (18) for the parameters β_1 and β_0 can be calculated as follows:

$$\frac{\partial SSE}{\partial \beta_0} = -2\sum_{i=1}^{n} [y_i(x_i) - \beta_0 - \beta_1 x_i] \tag{19}$$

$$\frac{\partial SSE}{\partial \beta_1} = -2\sum_{i=1}^{n} x_i[y_i(x_i) - \beta_0 - \beta_1 x_i] \tag{20}$$

After finding the first derivatives of the function, the first derivatives are equated with 0 to find the most optimal parameter that will minimizes the function. Thus, the following equation can be obtained:

$$\sum_{i=1}^{n} [y_i(x_i) - \beta_0 - \beta_1 x_i] = 0 \tag{21}$$

$$\sum_{i=1}^{n} x_i[y_i(x_i) - \beta_0 - \beta_1 x_i] = 0 \tag{22}$$

Through the results of first derivatives in equation (21), $\hat{\beta}_0$ can be obtained.

$$\sum_{i=1}^{n} y_i(x_i) - n\hat{\beta}_0 - \hat{\beta}_1 \sum_{i=1}^{n} x_i = 0 \tag{23}$$

That is,

$$\begin{aligned} n\hat{\beta}_0 &= \sum_{i=1}^{n} y_i(x_i) - \hat{\beta}_1 \sum_{i=1}^{n} x_i \\ \hat{\beta}_0 &= \frac{\sum_{i=1}^{n} y_i(x_i)}{n} - \frac{\hat{\beta}_1 \sum_{i=1}^{n} x_i}{n} \end{aligned} \tag{24}$$

Consider $\frac{\sum_{i=1}^{n} y_i(x_i)}{n}$ denoted by $\bar{y}$ as the average of $y_i(x_i)$ and $\frac{\sum_{i=1}^{n} x_i}{n}$ denoted by $\bar{x}$ as the average of x_i. Then, the final equation of $\hat{\beta}_0$ is as follows:

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x} \tag{25}$$

After that, through partial derivatives in equation (22) $\hat{\beta}_1$ can also be obtained.

$$\sum_{i=1}^{n} x_i y_i(x_i) - \hat{\beta}_0 \sum_{i=1}^{n} x_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i(x_i) - \bar{y}\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i^2 - \bar{x}\sum_{i=1}^{n} x_i}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i(x_i) - \bar{y}\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} (x_i - \bar{x})x_i}$$

Therefore, the final equation of $\hat{\beta}_1$ is obtained as follows:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i(x_i) - \bar{y}\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} (x_i - \bar{x})x_i} \tag{26}$$

E. Linear Least Squares Method On Gompertz Law Of Mortality

In the Gompertz distribution there are two parameters that must be estimated, namely B And c. These parameters are estimated using the LLS method. Therefore, the $y_i(x_i), \beta_1$ and β_0 will be sought from equation (17) with its application adapted to Gompertz's mortality law.

Through the force of mortality on Equation (7) we modified it using random variable T for each age $x+t$. The modified mortality function can be obtained as follows:

$$\mu_{x+t} = \frac{f_T(t)}{1-F_T(t)} \tag{27}$$

integrating into equation (27) we got,

$$\begin{aligned} \int \mu_{x+t}\, dt &= \int \frac{f_T(t)}{1-F_T(t)} dt \\ &= \int \frac{F_T'(t)}{1-F_T(t)} dt \\ &= -\ln\big(1-F_T(t)\big) \\ &= \ln\left(\frac{1}{1-F_T(t)}\right) \end{aligned} \tag{28}$$

Based on Equation (28) we know that Gompertz mortality function is $\mu_{x+t} = Bc^{x+t}$. Thus, we can write the following:

$$\begin{aligned} \ln\left(\frac{1}{1-F_T(t)}\right) &= \int \mu_{x+t}\, dt \\ &= \int Bc^{x+t}\, dt \\ &= \frac{Bc^x(c^t-1)}{\ln c} \end{aligned} \tag{29}$$

From equation (4) it stated $F_T(t) = {}_tq_x$, then equation (29) can also be written as the following:

$$\ln\left(\frac{1}{1-{}_tq_x}\right) = \frac{Bc^x(c^t-1)}{\ln c} \tag{30}$$

Since in general the information provided in the mortality table is q_x with $t=1$, then $t=1$ will be used in equation (30) which can be written as:

$$\begin{aligned} \ln\left(\frac{1}{1-q_x}\right) &= \frac{Bc^x(c-1)}{\ln c} \\ \ln\left\{\ln\left(\frac{1}{1-q_x}\right)\right\} &= \ln\left(\frac{Bc^x(c-1)}{\ln c}\right) \\ &= \ln\left(\frac{B(c-1)}{\ln c}\right) + x\cdot \ln c \end{aligned} \tag{31}$$

The equation (31) can now be assumed to be a simple linear regression as follows:

$$y_i(x_i) = \beta_0 + \beta_1 x_i$$

For $i = 1,2,\ldots,n$ with

$$y_i(x_i) = \ln\left\{\ln\left(\frac{1}{1-q_x}\right)\right\} \tag{32}$$

$$\beta_1 = \ln c \tag{33}$$

$$\beta_0 = \ln\left(\frac{B(c-1)}{\ln c}\right) \tag{34}$$

Where β_1 and β_0 is the estimate obtained from the equation (25) and (26).
The parameter B and c on Gompertz mortality function can now be found through equation (33) and (34) as following:

$$c = \exp\,(\hat{\beta}_1) \tag{35}$$

$$B = \frac{\exp\big(\hat{\beta}_0\big)\ln c}{c-1} \tag{36}$$

F. Discrete Whole Life Insurance

Discrete life insurance is insurance that is paid out at the end of the year of the Insured death. A model for life insurance in which the size and timing of death benefit payments depend only on the number of years the insured lives from the issuance of the policy to the time of death [3]. Discrete life insurance model is in terms of functions of the curtate-future-lifetime of the insured. The benefit is paid when the insured dies in year $k+1$ of insurance. Thus, the benefit function is denoted as b_{k+1} and the discount function denoted as v_{k+1}. The present value, on the issuance of benefit payment policies denoted by z_{k+1} is [3].

$$z_{k+1} = b_{k+1}v_{k+1} \tag{37}$$

Discrete whole life insurance is an insurance payable at the end of the year of death of the policy holder at any time in the future. The actuarial present value or whole life net single premium issued to (x) is [3].

$$A_x = \sum_{k=0}^{\infty} v^{k+1} \cdot {}_kp_x \cdot q_{x+k} \tag{38}$$

G. Discrete Whole Life Annuity

Discrete whole life annuity is an annuity that pays a unit amount each year that the insured (x) survives. The present value random variable Y, for such an annuity is given by $Y = \ddot{a}_{k+1}$ where the random variable K is the curtate future lifetime of (x). The probability associated with the value $\ddot{a}_{k+1}$ is $\Pr(K=k) = {}_kp_x \cdot q_{x+k}$. Thus, the actuarial present value of the annuity-due denoted by $\ddot{a}_x$ is [3]:

$$\ddot{a}_x = E[Y] = E[\ddot{a}_{k+1}] = E\left[\frac{1-v^{k+1}}{d}\right] \tag{39}$$

$$\ddot{a}_x = \sum_{k=0}^{\infty} \ddot{a}_{k+1} \cdot {}_kp_x \cdot q_{x+k} \tag{40}$$

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^k \cdot {}_kp_x \tag{41}$$

The equation (41) is the current payment form of the actuarial present value or a net single premium for a whole life annuity-due where the ${}_kp_x$ term is the probability of a payment of amount 1 being made at time *k*. If there is a limiting age then the life annuity-due value is calculated using the limited age. The limiting age is denoted by (ω) and the equation (41) is rewritten as follows:

$$\ddot{a}_x = \sum_{k=0}^{\omega-x-1} v^k \cdot {}_kp_x \tag{43}$$

H. Net Premium

Premium is a quantity of money that must be given to the insurance company by the policy holder in a predetermined manner and at the same time a condition for obtaining insurance coverage. Annual premium on insurance for life is the amount of costs borne by the participants insurance paid annually in order to obtain benefits when the policy holder die. The net premium is computed using only the benefits and premiums patterns and do not consider expenses, profit or contingency margins [3]. The benefit premium has three principles. Principle I are known as percentile premiums, principle 2 is called the equivalence principal with the requirement of $E[L] = 0$, equivalently,

$$E[L] = E[present\ value\ of\ benefits] - E[present\ value\ of premiums]$$

From the equation above yields,

$$\begin{aligned} E[L] &= E[Z] - E[Y] \\ 0 &= E[v^{k+1}] - P_x \cdot E[\ddot{a}_{k+1}] \\ 0 &= A_x - P \cdot \ddot{a}_x \\ P &= \frac{A_x}{\ddot{a}_x} \end{aligned} \tag{44}$$

The premium can be obtained by dividing the present value of benefits (life insurance) to the present value of premiums (life annuity).

III. Method

A. Data Collection Design

This study uses secondary data, namely the Indonesian Mortality Table (TMI) version IV released on 2019 [8].

B. Flowchart Process

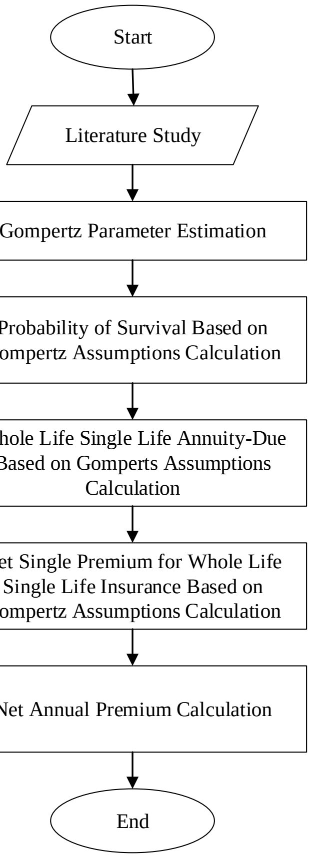

Figure 1. Flowchart Process

The steps of calculating the net annual premium of whole life single life insurance using Gompertz mortality law starts with conducting the literature study, in the literature study, all of the formula that will be used in calculating the net annual premium will be known. After conducting the literature study, the Gompertz parameter estimation will be calculated and all of the formula and method used is on the literature study that have been previously conducted. Next, computing the probability of survival based on Gompertz mortality law and used it for computing the life annuity-due value. Then, compute the net single premium of whole life single life insurance based on Gompertz mortality law. Lastly, compute the net annual premium

IV. RESULT AND DISCUSSION

A. Gompertz Mortality Law Parameter Estimation with Linear Least Squares (LLS) Method

The Gompertz parameters will be estimated from the Indonesian Mortality Table (TMI IV 2019) using the Linear Least Squares (LLS) parameter estimation method. The calculation of the net annual premium is for woman and man aged 30 years. The interest rate used is a fixed interest rate of 5.75% based on Bank Indonesia 7-Day Reverse Repo Rate at March 2023. With the total benefits assumed IDR 100,000,000.

By using data on TMI IV for woman and man each, the value of $y_i(x_i)$ can be obtained from equation (32). The estimates of β_1 and β_0 can be obtained and calculated through equations (25) and (26). The calculation of β_1 conducted using Excel is as follows:

For woman:	For man:
$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i(x_i) - \bar{y}\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n}(x_i - \bar{x})x_i} = \frac{\sum_{i=1}^{111} x_i y_i(x_i) - \bar{y}\sum_{i=1}^{111} x_i}{\sum_{i=1}^{111}(x_i - \bar{x})x_i} = \frac{9582.42766}{120176} = 0.07974$	$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i(x_i) - \bar{y}\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n}(x_i - \bar{x})x_i} = \frac{\sum_{i=1}^{111} x_i y_i(x_i) - \bar{y}\sum_{i=1}^{111} x_i}{\sum_{i=1}^{111}(x_i - \bar{x})x_i} = \frac{9559.432}{120176} = 0.079545$

Continuing, the calculation of β_0 for woman and man are as follows:

For woman:	For man:
$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x}$ $= -5.16155 - (0.07974 \cdot 55)$ $= -9.58693$	$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x}$ $= -4.84650 - (0.079545 \cdot 55)$ $= -9.22149$

Hence, the Gompertz Law of Mortality parameters *B* and *c* could be calculated. The parameters value is calculated using the equation from (33) and (34). The parameter *c* value for man and woman are calculated as follows:

For woman:	For man:
$c = \exp(\hat{\beta}_1)$ $= \exp(0.07974)$ $= 1.083$	$c = \exp(\hat{\beta}_1)$ $= \exp(0.079545)$ $= 1.082795$

Continuing, the parameter B value for woman and man are calculated as follows:

For woman:	For man:
$B = \frac{\exp(\hat{\beta}_0) \ln c}{c - 1}$ $= \frac{\exp(-9.58693) \ln(1.083)}{1.083 - 1}$ $= 0.00006592$	$B = \frac{\exp(\hat{\beta}_0) \ln c}{c - 1}$ $= \frac{\exp(-9.22149) \ln(1.082795)}{1.082795 - 1}$ $= 0.00009501$

B. Probability of Survival and Death Age x

Probability of survival (p_x) based on the Gompertz mortality law can be calculated using the formula in equation (17). Furthermore, after calculating the probability of survival the probability of death can be calculated using the formula in equation (7). The probability of death (q_x) is used to find the net single premium of life insurance. The probability of survival and death for *x* age 30 for woman is expressed in table 1 and for man is expressed in table 2.

TABLE 1
PROBABILITY OF SURVIVAL AND DEATH FOR WOMAN AGE X

x	p_x	q_x
30	0.991819	0.008181
31	0.991075	0.008925
32	0.99027	0.009730
33	0.989399	0.010601
34	0.988456	0.011544
35	0.987436	0.012564
36	0.986333	0.013667
.	.	.
.	.	.
.	.	.
108	0.010661	0.989339
109	0.007313	0.992687
110	0.004862	0.995138

TABLE 2
PROBABILITY OF SURVIVAL AND DEATH FOR MAN AGE X

x	p_x	q_x
30	0.988276	0.011724
31	0.987214	0.012786
32	0.986065	0.013935
33	0.984822	0.015178
34	0.983479	0.016521
35	0.982026	0.017974
36	0.980456	0.019544
.	.	.
.	.	.
.	.	.
108	0.001616	0.998384
109	0.000949	0.999051
110	0.000533	0.999467

C. Probability of Survival and Death Age (x+k)

Probability of survival (${}_kp_x$) based on the Gompertz mortality law can be calculated using the formula in equation (18). Furthermore, after calculating the probability of survival the probability of death can be calculated using the formula in equation (7). Since there is a limiting age of 111 in the mortality table (TMI IV) the probability of survival and death is also calculated based on the limiting age for a woman and man age 30 years.

The calculation of ${}_kP_{30}$ and ${}_kq_{30}$ are calculated using Excel and the values for woman is described in table 3 and for man is describe in table 4.

TABLE 3
PROBABILITY OF SURVIVAL AND DEATH FOR WOMAN AGE X+K

k	${}_kp_{30}$	${}_kq_{30}$
0	1	0
1	0.99925	0.00075
2	0.99844	0.00156
3	0.99756	0.00244
4	0.99661	0.00339
5	0.99558	0.00442
6	0.99447	0.00553
.	.	.
.	.	.
.	.	.
78	0.01075	0.98925
79	0.00737	0.99263
80	0.00490	0.99510

TABLE 4
PROBABILITY OF SURVIVAL AND DEATH FOR MAN AGE X+K

k	${}_kp_{30}$	${}_kq_{30}$
0	1	0
1	0.99893	0.00108
2	0.99776	0.00224
3	0.99651	0.00349
4	0.99515	0.00485
5	0.99368	0.00632
6	0.99209	0.00791
.	.	.
.	.	.
.	.	.
78	0.00164	0.99837
79	0.00096	0.99904
80	0.00054	0.99946

D. Whole Life Single Life Annuity Due Based on Gompertz

The calculation of the discount factor (v) is computed using the discount factor formula as on equation (The interest rate used is a fixed interest rate of 5.75% based on Bank Indonesia 7-Day Reverse Repo Rate per March 2023. The calculation of the discount factor is as follows:

$$v = \frac{1}{1+i}$$
$$= \frac{1}{1+0.0575}$$
$$= 0.945626$$

The calculation of discrete whole life singe life annuity-due for woman age 30 years old will be described as follows:

$$\ddot{a}_{30} = \sum_{k=0}^{80} v^k \cdot {}_kp_{30}$$
$$= (v^0 \cdot {}_0p_{30}) + (v^1 \cdot {}_1p_{30}) + (v^2 \cdot {}_2p_{30}) + \cdots + (v^{80} \cdot {}_{80}p_{30})$$
$$= (1 \cdot 1) + (0.94563 \cdot 0.99925) + \cdots + (0.01142 \cdot 0.004902)$$
$$= 1 + 0.944917 + \cdots + 0.000056$$
$$= 16.93589$$

The complete calculation of the whole life single life annuity-due value based on the Gompertz mortality law for a woman age 30 years old are described in table

TABLE 5
WHOLE LIFE ANNUITY DUE FOR WOMAN AGE 30

k	v^k	${}_kp_{30}$	$v^k \cdot {}_kp_{30}$
0	1	1	1
1	0.945626	0.99925	0.944917
2	0.894209	0.998438	0.892813
3	0.845588	0.99756	0.843525
4	0.799611	0.996609	0.796899
5	0.756133	0.995581	0.752792
6	0.715019	0.994469	0.711064
7	0.676141	0.993265	0.671587
.	.	.	.

.	.	.	.
.	.	.	.
78	0.012768	0.010749	0.000137
79	0.012074	0.007373	0.000089
80	0.011417	0.004902	0.000056
Total Summation			16.93589

It can be concluded from table 5 that the whole life single life annuity-due value for woman age 30 years old based on Gompertz mortality assumptions is 16.93589.

The calculation of discrete whole life singe life annuity-due for a man age 30 years old will be described as follows:

$$\begin{aligned}\ddot{a}_{30} &= \sum_{k=0}^{80} v^k \cdot {}_kp_{30} \\ &= (v^0 \cdot {}_0p_{30}) + (v^1 \cdot {}_1p_{30}) + (v^2 \cdot {}_2p_{30}) + \cdots + (v^{80} \cdot {}_{80}p_{30}) \\ &= (1 \cdot 1) + (0.94563 \cdot 0.99893) + \cdots + (0.01142 \cdot 0.00054) \\ &= 1 + 0.94461 + \cdots + 0.0000062 \\ &= 16.59562\end{aligned}$$

The complete calculation of the whole life single life annuity-due value based on the Gompertz mortality law for a man age 30 years old are described in table 6

TABLE 6
WHOLE LIFE ANNUITY DUE FOR MAN AGE 30

k	v^k	${}_kp_{30}$	$v^k \cdot {}_kp_{30}$
0	1	1	1
1	0.945626	0.998925	0.94461
2	0.894209	0.997763	0.892209
3	0.845588	0.996506	0.842633
4	0.799611	0.995146	0.795729
5	0.756133	0.993676	0.751351
6	0.715019	0.992087	0.709361
7	0.676141	0.990369	0.669629
.	.	.	.
.	.	.	.
78	0.012768	0.001635	0.000021
79	0.012074	0.00096	0.000012
80	0.011417	0.00054	0.0000061
Total Summation			16.59562

It can be concluded from table 6 that the whole life single life annuity-due value for man age 30 years old based on Gompertz mortality assumptions is 16.59562, smaller than the value for woman.

E. Whole Life Single Life Net Single Premium Based on Gompertz

The calculation of net single premium for woman will be conducted through the formula in equation (40). It is as follows:

$$\begin{aligned}A_{30} &= \sum_{k=0}^{80} v^{k+1} \cdot {}_kp_x \cdot q_{x+k} \\ &= (v^1 \cdot {}_0p_{30}q_{30}) + (v^2 \cdot {}_1p_{30}q_{31}) + \cdots + (v^{81} \cdot {}_{80}p_{30}q_{110}) \\ &= (0.94563 \cdot 1 \cdot 0.008181) + \cdots + (0.010797 \cdot 0.004902 \cdot 0.995138) \\ &= 0.814444329\end{aligned}$$

Therefore, the net single premium based on Gompertz mortality assumptions for woman aged 30 years old with 5.75% interest rate is 0.814444329.

The calculation of net single premium for man will be conducted through the formula in equation (40). It is as follows:

$$A_{30} = \sum_{k=0}^{80} v^{k+1} \cdot {}_{k}p_x \cdot q_{x+k}$$
$$= (v^1 \cdot {}_{0}p_{30}q_{30}) + (v^2 \cdot {}_{1}p_{30}q_{31}) + \cdots + (v^{81} \cdot {}_{80}p_{30}q_{110})$$
$$= (0.94563 \cdot 1 \cdot 0.011724) + \cdots + (0.010797 \cdot 0.00054 \cdot 0.999467)$$
$$= 0.999128755$$

Therefore, the net single premium based on Gompertz mortality assumptions for man aged 30 years old with 5.75% interest rate is 0.999128755.

F. Whole Life Single Life Net Annual Premium Based on Gompertz

The calculation of whole life single life insurance net annual premium for woman will be conducted through the formula in equation (45) with the benefit of IDR 100,000,000. It is as follows:

$$P = \frac{100{,}000{,}000 \cdot A_{30}}{\ddot{a}_{30}}$$
$$= \frac{(100{,}000{,}000) \cdot (0.814444329)}{16.93589168}$$
$$= \text{IDR } 4{,}808{,}984.04$$

Therefore, it is concluded that the net premium based on Gompertz mortality assumptions for a woman aged 30 years old with 5.75% interest rate and benefit assumed of IDR 100,000,000 that must be paid annually until death is IDR 4,808,984.04.

The calculation of whole life single life insurance net annual premium for man will be conducted through the formula in equation (2.61) with the benefit computed. It is as follows:

$$P = \frac{100{,}000{,}000 \cdot A_{30}}{\ddot{a}_{30}}$$
$$= \frac{(100{,}000{,}000) \cdot (0.999128755)}{16.59561851}$$
$$= \text{IDR } 6{,}020{,}436.98$$

Therefore, it is concluded that the net premium based on Gompertz mortality assumptions for a man aged 30 years old with 5.75% interest rate and benefit assumed of IDR 100,000,000 that must be paid annually until death is IDR 6,020,436.98.

V. CONCLUSION

From this research, it can be concluded that the calculation of the premium for whole life single life insurance using Gompertz mortality assumptions is influenced by parameters on the Gompertz distribution, the interest rate used, and the Insurer age. According to the results of the research conducted, the value of the premium differs from man and woman with the same age of 30 years old, same interest rate and benefit function of IDR 1,000,000,000. The difference of the premium is caused by the difference in the probability of death from the mortality table that is used to compute the Gompertz parameter estimation using the Linear Least Squares Error (LLS) method. The value of the premium based on Gompertz mortality assumptions using Linear Least Squares (LLS) method for man is higher than the value of the premium for woman for whole life insurance.

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