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Estimation of Premium Reserve for Last Survivor Endowment Insurance Using the New Jersey Method

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Abstract— There are a few cases of life insurance firms going bankrupt due to mistakes when estimating premium reserves, causing companies unable to pay compensation to policyholders. This is caused when the number of claims submitted by the insured that must be paid exceeds the number of claims previously estimated. Situations like this can be anticipated if the insurance firm has a properly prepared and calculated reserve value. There are various types of life insurance, one of which is endowment life insurance. The purpose of this study is to calculate the amount of reserves adjusted for last survivor endowment life insurance using the New Jersey method and the Indonesian Mortality Table (TMI) 2019. To compute reserves, first, researchers determine the benefits, then the annuity, and finally the annual premium. In the first year, the premium reserve value under the New Jersey method is zero. The New Jersey method begins premium reserves in the second year, for t years, with $t = 2, 3, \dots, n$ where n reflecting the duration of the insurer's contract. Based on the data analysis performed, the value of the New Jersey reserves for the two insureds is determined by the length of the insurance contract and the age of the insured at the time of insurance. The value of premium reserves using the New Jersey method from the end of year 1 to the end of year 50 is always rising. The value of reserves is seen based on the initial age of a person when starting insurance, the older the initial age of the insurance participant, the greater the amount of reserves obtained by the company. This is due to the higher death rate in the elderly age. This research is expected can be a reference and help insurance field to estimate premium reserves.

Keywords— *Endowment Life Insurance, Premium Reserves, Last Survivor, New Jersey Method*

I. INTRODUCTION

Human life cannot be separated from unpredictable future events, such as natural disasters, accidents, disease, and even death. Unwanted events can happen to anyone, anywhere, and at any moment. Therefore, humans need protection that can overcome these risks, one of which is insurance. Insurance is one of the keys to minimizing the impact of unfavorable events. Insurance is a contract between two parties in which the first party (the insured) agrees to pay contributions to the second party (the insurer) in accordance with a written agreement and the second party is required to provide the insured with the full guarantee if something unfavorable occurs. One of the certain risks faced is death, which cannot be predicted in terms of timing or reason. Death can be caused by illness or accident. Indonesia has various types of insurance. Insurance that protects a person's life is called life insurance. According to the Asosiasi Asuransi Jiwa Indonesia (AAJI), life insurance is required to ensure that economic necessities are not risked due to risks to breadwinners during productive times [1].

Life insurance protects the insured by transferring financial losses caused by death. If there is a risk of death within the time period agreed in the insurance contract, the heirs will receive an amount promised or compensation as a benefit of the insurance [2]. Life insurance is classified into three types based on the length or period of coverage and the number of insured, namely term life insurance, whole life insurance, and endowment life insurance [3]. There are two types of life insurance based on the number of insured, namely individual life insurance (single life) and multi life. Multi life insurance has two terms based on the death status of the insured group, namely joint life and last survivor [4]. The difference between joint life insurance and last survivor life insurance is in the method of premium payment. Premium payments on joint life insurance are made until one of them dies, at which point the insurer pays compensation. Whereas premium payments for last survivor life insurance in the case of two insured persons are paid until the two insureds die, whether they die simultaneously or not, and compensation is provided from the insurer at that time. The last survivor insurance case uses the last survivor annuity. A last survivor annuity is a series of payments made by two or more insured people in which premium payments are made only if one of the insured is still alive at the end of the insurance period.

An insured who follows an insurance policy must pay a premium that has been previously agreed upon with the insurance company. There are two types of premiums, namely net premiums and gross premiums. The insurance company will allocate the premium payments made by insurance participants for reserve value, company operations, and compensation [5]. Insurance participants or the insured can make a claim if uncertain events occur during the insurance period. This might also cause losses and risk for the insurance company. Most life insurance companies suffer losses due to unexpected claims, policyholder termination of premium payments for any reason, and unpredictable company expenses. As a result of this issue, Otoritas Jasa Keuangan (OJK) has announced the liquidation of several insurance firms on its website in recent years.

In insurance companies, the payments paid when an insurance member dies at a specified time are taken from the premium reserve. Premium reserves are the funds held by the corporation during the coverage period. Premium reserves are critical for insurance firms because they are used to provide compensation that will be returned to the insured or can be used in the event of a claim [6]. The insurance company definitely has costs that must be incurred, so the estimation of premium reserves needs to consider these costs. If funds are not properly managed, this incident potentially leads to financial losses for the insurance firm. This problem can be solved if the insurance company has a reserve fund that has been properly prepared and calculated correctly. The determination of the reserve value is influenced by several factors, including the age of the insurance participant, the amount of compensation, the interest rate, and the length of the premium payment [5]. Calculation of premium reserves is divided into two, namely retrospectively and prospectively. Retrospective calculations are based on total income from the past until the calculation of reserves is carried out, reduced by the amount of expenditure in the past for each insurance participant. Meanwhile, prospective calculations are based on the present value of all expenses in the future minus the present value of total income in the future for each insurance participant [3].

Calculation of premium reserves needs to be adjusted so that the company gets a new source of funds to cover costs in the initial year of the policy and prevent losses at the beginning of the year for calculating premium reserves. These funds can later be considered as loans that will be paid later from gross premium payments in the coming years. There are various calculation methods used to modify premium reserves prospectively, one of them is the New Jersey method. The New Jersey method was chosen in this study to calculate the amount of premium reserves that must be prepared by a company. The New Jersey method is a method that determines that the reserve value at the end of the first year is zero, which is useful to cover the shortfall in costs in the first year of the policy so that the premiums paid by insurance participants can be used by the company for the company's operational costs, then will be paid back from the premiums for the following years. The New Jersey method is a method that was developed as an improvement to the Illinois approach, where the premium payments that exceed 20 times the payment on the New Jersey method produce a more effective reserve fund. The purpose of this study is to calculate the premium reserve that must be prepared by insurance companies for endowment life insurance products using the New Jersey method.

The New Jersey method of estimating premium reserves has previously been discussed by other researchers. Premium reserves using the New Jersey method and Full Preliminary Term on endowment insurance concluded that the premium reserve value of the New Jersey Method is always greater than the value of the Full Preliminary Term Method premium reserve every year, except in the first and last years [7]. The estimation of monthly premium reserves for last survivor whole life insurance using the New Jersey method, and the value of the resulting premium reserve is the same every month until one of the insured's aged x or y dies. However, when one of the insureds who is x years old or y years old dies, the premium reserve value 18 increases quite significantly [8]. The annual premium for last survivor term life insurance depends on the interest rate, gender and age of the insured. Males pay higher premiums than females. The higher the interest rate, the smaller the premium paid, while the premium is more expensive if the age of the customer is higher [9]. Previous researchers applied the New Jersey method by using joint life insurance and individual life insurance. As a result, it is still rare to calculate the amount of the premium reserve with the last survivor endowment life insurance case. It was stated that, based on the number of insureds are cases of join life, single life, and last survivor for whole life insurance. In this study, researchers will use endowment life insurance for the last survivor case using the New Jersey method, so that it can be determined whether way is more effective implementing these calculations.

The study focused on endowment insurance products in the last survivor cases by estimating the prospective reserve value with the new jersey method and the Indonesian Mortality Table (TMI) 2019. In the calculation of the new jersey method, assuming that the payment of contributions will continue to run as long as there is still one insured person who is still alive. Payment of contributions will end when the last member of this insurance group dies. This method is used for insurance with premiums that exceed 20 times the payment, and the reserve value at the end of the first year is zero, so the premium paid by insurance participants can be used by the company to cover the company's operational costs [10]. To calculate the premium reserve value using the New Jersey method, first determine the insurance participant's age (xy) and the period of coverage (n). Understanding the chance of survival, the chance of someone dying which is presented in the mortality table, the interest rate, and the amount of compensation that insurance participants will receive. Based on the description above, the author raised the title of this study, namely estimating the annual premium reserve for last survivor endowment insurance using the New Jersey prospective method.

II. LITERATURE REVIEW

A. Interest

The term of interest is important in calculating a life insurance premium because if someone invests a fund within a certain period of time, the fund will grow to cover the amount of claim or compensation money that will be issued by the company. Interest is the amount paid by the borrower to the lender in consideration for the use of money.

The effective interest rate (i) is the ratio of the amount of interest earned during a certain period to the amount of principal at the beginning of the period. And the effective discount rate (d) is the ratio of the amount of discount obtained during a given period to the amount of accumulated value at the end of the period, where d is written as

$$d = \frac{i}{1+i} \tag{1}$$

The present value is a 1 investment that accumulates to $1 + i$ at the end of period 1. The present value can also be called the discount factor which is indicated by v and can be expressed as

$$v = \frac{1}{1+i} \tag{2}$$

There are two types of interest rate calculations, namely simple interest and compound interest [3]. Simple interest is a calculation of the value of interest that only considers the amount of initial capital invested as compared to the length of the investment. The simple interest of the initial capital P for n periods with an interest rate of i can be denoted as follows:

$$I = Pni \tag{3}$$

and the sum accumulated for n period is

$$P_n = P + I = P + Pni = P(1 + ni) \tag{4}$$

Compound interest is the calculation of the principal amount of interest where the investment term is the sum of the previous principal amount and the amount of interest earned [3]. For example, if the amount of principal *(P)* is invested with interest (*i)* during time period (*n*), the compound interest formula is as follows:

$$P_n = P(1+i)^n \tag{5}$$

where p_n is accumulated amount, I is amount of interest.

Interest rates are typically stated in one year (annually) and distributed over a certain period. This interest rate is called the annual nominal interest rate. The nominal and effective interest rates are represented in percent per year and are denoted by i for the effective interest rate and $i^{(m)}$ for the nominal interest rate where m is the number of interest distribution periods. So that cumulatively there is a relationship between the nominal interest rate and the effective interest rate as follows [3]:

$$1 + i = \left[1 + \frac{i^{(m)}}{m}\right]^m$$

$$1 + \frac{i^{(m)}}{m} = (1+i)^{1/m}$$

$$i^{(m)} = m\left[(1+i)^{\frac{1}{m}} - 1\right] \tag{6}$$

B. Survival Distribution

Suppose X is a continuous variable that expresses the age until the death of a birth. If $F_x(x)$ is the distribution function of X, then [4]:

$$F_x(x) = Pr(X \le x), \quad x \ge 0 \tag{7}$$

this means the probability that a person will die before reaching age x, then, the survival function $S(x)$ is defined as the probability that a person will survive to reach age x [4], namely:

$$S(x) = \Pr(X > x), \quad x \ge 0$$
$$S(x) = 1 - F_x(x), \quad x \ge 0 \tag{8}$$

the function of $S(x)$ is known as the survival function. The probability that a person is born and then dies at the age of 0 is 0. $F(0)$ = 0, then $S(0)$ = 1 is obtained, meaning that the probability that someone born will still be alive at the age of 0 years = 1. The probability of a newborn dying between the ages of x and z, if given $x < z$, with the equation is:

$$\begin{aligned} Pr(x < X \leq z) &= F_x(z) - F_x(x) \\ &= S(x) - S(z) \end{aligned} \tag{9}$$

the conditional probability that a person aged x will die between the ages of x and z, $z > x$ can be determined using the following formulation:

$$\begin{aligned} Pr(x < X \leq z | X > x) &= \frac{P(x < X \leq z)}{P(X > x)} \\ &= \frac{F_x(z) - F_x(x)}{1 - F_x(x)} \\ &= \frac{F_x(z) - F_x(x)}{S(x)} \end{aligned}$$

or can be written as,

$$Pr(x < X \leq z | X > x) = \frac{S(x) - S(z)}{S(x)} \tag{10}$$

Let $T(x)$ be a random variable that represents a person's life time calculated from age x. Then the probability that a person aged x will die in t the coming year, can be written with the notation ${}_tq_x$ which is a function distribution of $T(x)$.

$${}_tq_x = Pr(T(x) \leq t), t \geq 0. \tag{11}$$

The probability that a person aged x will survive in t years more or will survive until $(x + t)$ years, can be denoted by ${}_tp_x$ which is the survival function for age x

$$\begin{aligned} {}_tp_x &= Pr(T(x) > t), t \geq 0 \\ &= 1 - {}_tq_x \end{aligned} \tag{12}$$

in the special case at age 0, namely x = 0 or $T(0) = x$, obtained:

$${}_tp_0 = S(x), \qquad x \geq 0 \tag{13}$$

based on equation (12) will be obtained,

$${}_tp_x = \frac{{}_{x+t}p_0}{{}_tp_0} = \frac{S(x+t)}{S(x)} \tag{14}$$

based on equation (11) will be obtained,

$$\begin{aligned} {}_tq_x &= 1 - {}_tp_x = 1 - \frac{S(x+t)}{S(x)} \\ {}_tq_x &= \frac{S(x) - S(x+t)}{S(x)} \end{aligned} \tag{15}$$

In the discrete case, the random variable indicating the remaining age the discrete time of x is denoted by K as curtate-future-lifetime and states the total number of years lived by someone before death. The variables K and T have relationship that is, K has the value of the largest integer that less than or equal to the value of T, we define as $K = |T|$. The random variable K has a probability function denoted with Pr $(K = k)$. Due to the relationship between K and T then there is a relationship between the probability function of K and the function distribution and survival function of T. Here's the breakdown:

$$\begin{aligned} \Pr(K = k) &= \Pr\,(k \leq T < k + 1) \\ &= \Pr\,(k < T \leq k + 1) \\ &= \Pr(T \leq k + 1) - \Pr\,(T < k) \end{aligned}$$

based on equation (15) obtained the following equation:

$$\begin{aligned} \Pr(K = k) &= {}_{k+1}q_x - {}_kq_x \\ &= (1 -_{k+1} p_x) - (1 -_k p_x) \\ &= {}_kp_x - {}_{k+1}p_x \\ &= \frac{S\,(x+k)}{S\,(x)} - \frac{S\,(x+k+1)}{S\,(x)} \end{aligned}$$

$$= \left(\frac{S(x+k) - S(x+k+1)}{S(x)}\right) . \frac{S(x+k)}{S(x+k)}$$
$$= \frac{S(x+k)}{S(x)} . \left(\frac{S(x+k)}{S(x+k)} - \frac{S(x+k+1)}{S(x)}\right)$$
$$= {}_kp_x(1 - p_{x+k})$$
$$= {}_kp_x . q_{x+k} \quad (16)$$

C. Mortality Table

A mortality table is a summary table of a report that describes a number of individual groups [13]. Mortality tables (death tables) are very important in annuity and life insurance calculations. This table is arranged based on mathematical formulas and probability.

The mortality table consists of several columns which consist of the first column, namely x for the age of the participants, the second column, namely l_x for the exact number of people aged x years, the third column is d_x for the number of people who died aged x years, the fourth column is q_x for someone who is x years old dies before the age of $x + 1$ year old, column fifth, namely p_x for the life chances of someone aged x years, and column the sixth is e_x for the life expectancy of someone aged x years [12].

the basic relationship used based on the above term i.e

$$d_x = l_x - l_{x+1} \quad (17)$$

d_x is the number of people who die before reaching age x+1, and

$$l_x = d_x + d_{x+1} + \ldots + d_{x+n-1} + d_{x+n} \quad (18)$$

l_x is the number of life people during age x until $x + 1$ years.

While the p_x and q_x formulas are as follows

$$p_x = \frac{l_{x+1}}{l_x} \quad (19)$$

p_x is the probability of life of a people aged x year, and

$$q_x = 1 - p_x = 1 - \frac{l_{x+1}}{l_x} = \frac{l_x - l_{x+1}}{l_x} = \frac{d_x}{l_x} \quad (20)$$

q_x is the probability of death of a person aged x year.

The following are the formulas related to the probability of living and the value of the probability of dying, the symbol (x) means a person aged x. The probability value (x) of living years (${}_tp_x$) is

$${}_tp_x = \frac{l_{x+t}}{l_x} \quad (21)$$

the probability value (x) of death within years (${}_tq_x$) is

$${}_tq_x = \frac{l_x - l_{x+t}}{l_x} = 1 - {}_tp_x \quad (22)$$

furthermore, the probability of death of a policyholder aged x year will die before the age of $x+t$ years is stated by

$${}_tq_x = \frac{{}_td_x}{l_x} \quad (23)$$

where ${}_td_x$represents the number of people who died between the ages of x years and $x+t$ years expressed by

$${}_td_x = l_x - l_{x+t} \quad (24)$$

D. Commutation Symbols

The commutation table's calculations are closely related to those of the mortality table [3]. The commutation table symbols are used to calculate annual premiums, premium reserves, and other insurance values. In the actuarial calculation, commutation symbols are often used. The commutation symbol aims to simplify the writing of formulas in calculations. These are the symbols:

D_x is the commutation symbol derived from the product of the cash value of the payment (v) to the power of age x years with the number of insurance participants living at the age of x years, indicates as:

$$D_x = v^x . l_x \quad (25)$$

N_x is the commutation symbol for the accumulated value of D_{x+k} with $k = 0$ years up to w, denoted as:

$$N_x = \sum_{k=0}^{w} D_{x+k} \quad (26)$$
$$= D_x + D_{x+1} + \cdots + D_w$$

C_x is a commutation symbol from the result of multiplying the cash value of payments (v) to the rank of age x years with the number of insurance participants who died at the age of x years, denoted as:

$$C_x = v^{x+1}.\, d_x \tag{27}$$

M_x is the commutation symbol for the accumulated value of C_{x+k} with $k = 0$ years up to w, denoted as:

$$\begin{aligned} M_x &= \sum_{k=0}^{w} C_{x+k} \\ &= C_x + C_{x+1} + \cdots + C_w \end{aligned} \tag{28}$$

w indicates the highest age until the insurance term.

E. Annuity Due

Annuity is a deposit that must be paid continuously by a customer in a certain amount and at predetermined time intervals [3]. Annuities can be divided into two types, namely fixed annuities and life annuities. A fixed annuity is an annuity where the payment of benefits does not depend on the death of a person, meaning that payments must be provided even if the insured person dies or is still alive. An annuity life is an annuity in which payments are given based on the life and death of a person. Based on the method of payment, life annuities are divided into two types, namely discrete annuities and continuous annuities. Discrete annuity means that the annuity payments are made periodically, such as monthly, quarterly, semiannually, or annually. If payments m times a year can be paid at any time so that $m \to \infty$ and the number of annual payments is 1 unit, then it is called a continuous annuity. Payments can be made at the beginning of each year, which is called annuity-due.

The annuity-due in which payments are made at the beginning of the period instead. The present value of the annuity-due is denoted by $\ddot{a}_{\overline{n|}}$ and the accumulated value of the annuity-due is denoted by $\ddot{s}_{\overline{n|}}$. An expression for $\ddot{a}_{\overline{n|}}$ can be written as follows:

$$\ddot{a}_{\overline{n|}} = 1 + v + v^2 + \ldots + v^{n-1}$$

summing the geometric progression

$$\begin{aligned} \ddot{a}_{\overline{n|}} &= \frac{1 - v^n}{1 - v} \\ \ddot{a}_{\overline{n|}} &= \frac{1 - v^n}{iv} \\ \ddot{a}_{\overline{n|}} &= \frac{1 - v^n}{d} \end{aligned} \tag{29}$$

for $\ddot{s}_{\overline{n|}}$, the following formulas can write by

$$\ddot{s}_{\overline{n|}} = (1+i) + (1+i)^2 + \ldots + (1+i)^{n-1} + (1+i)^n$$

summing the geometric progression

$$\begin{aligned} \ddot{s}_{\overline{n|}} &= (1+i)\,\frac{(1+i)^n - 1}{(1+i) - 1} \\ \ddot{s}_{\overline{n|}} &= \frac{(1+i)^n - 1}{iv} \\ \ddot{s}_{\overline{n|}} &= \frac{(1+i)^n - 1}{d} \end{aligned} \tag{30}$$

F. Life Term Annuity Due

The cash value of a life annuity due is calculated at the beginning of each term of the annuity receipt. The annuity due and annuity immediate cash values differ by 1, namely the lifetime annuity due payment is one year earlier than the immediate lifetime annuity [3]. The present value random variable of an n-year temporary life annuity-due of one per years indicated by Y and can be expressed as:

$$Y = \begin{cases} \ddot{a}_{\overline{k+1|}} & ,0 \le k < n \\ \ddot{a}_{\overline{n|}} & ,k \ge n \end{cases} \tag{31}$$

where k is a random discrete variable representing the future lifetime of an aged x, thus, $\ddot{a}_{x:\overline{n|}}$ can be determined by the following formulation:

$$\begin{aligned} \ddot{a}_{\overline{x:n|}} = E[Y] &= 1 + v\, p_x + v^2\, {}_2p_x + \ldots + v^{n-1}\, {}_{n-1}p_x \\ &= \sum_{k=0}^{n-1} v^k \cdot {}_kp_x \end{aligned}$$

by using the commutation symbol described in section 4, the cash value of the initial futures annuity is calculated with the following formula

$$\ddot{a}_{x:\overline{n|}} = a_{\;x:\overline{n-1|}} + 1$$
$$\ddot{a}_{\overline{x:n|}} = 1 + \frac{v^{x+1}l_{x+1} + v^{x+2}l_{x+2} + \cdots + v^{n-1}l_{x+n-1}}{v^x l_x}$$
$$\ddot{a}_{\overline{x:n|}} = \frac{v^x l_x + v^{x+1}l_{x+1} + v^{x+2}l_{x+2} + \cdots + v^{n-1}l_{x+n-1}}{v^x l_x}$$
$$\ddot{a}_{\overline{x:n|}} = \frac{D_x + D_{x+1} + \cdots + D_{x+n-1}}{D_x} \quad (32)$$
$$\ddot{a}_{\overline{x:n|}} = \frac{N_x - N_{x+n}}{D_x}$$

G. N-year Term Life Insurance

Term insurance policy with a term of n years pays out only if the policyholder dies during that time period [11]. For example, 1 unit is awarded only if the death occurs within the first n years, and payment is made at the end of the year in which the death occurs. N-year term life insurance by providing one unit of benefit (b_{k+1}) at the end of the year of death, obtained:

$$b_{k+1} = \begin{cases} 1 & , k = 0,1,2,\dots,n-1 \\ 0 & , k = n, n+1, \dots \end{cases}$$
$$v_{k+1} = \begin{cases} v^{k+1} & , k = 0,1,2,\dots,n-1 \\ 0 & , k = n, n+1, \dots \end{cases}$$
$$Z = \begin{cases} v^{k+1} & , k = 0,1,2,\dots,n-1 \\ 0 & , k = n, n+1, \dots \end{cases}$$

with *Z* is random variable for present value of benefit (Bowers, Gerber, Hickman, & Jones, 1997):

$$Z_{k+1} = b_{k+1}.\, v^{k+1}$$
$$Z_{k+1} = 1.\, v^{k+1} = v^{k+1} \quad (33)$$

then the Actuarial Present Value (APV) of an aged x years for n-year term benefit is denoted $A^1_{x:\overline{n|}}$ so that:
$A^1_{x:\overline{n|}} = E[Z] = \sum_{k=0}^{n-1} \Pr\,(K = k)$

$$A^1_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1}.\, {}_kp_x.\, q_{x+k}$$

by replacing ${}_kp_x.\, q_{x+k}$ by q_{x+k}/l_x, then obtained $A^1_{x:\overline{n|}}$ using the commutation symbol

$$A^1_{x:\overline{n|}} = \frac{v^x d_x + v^{x+1}d_{x+1} + v^{x+2}d_{x+2} + \cdots + v^{n-1}d_{x+n-1}}{l_x}$$
$$A^1_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{D_x} \quad (34)$$

H. Pure Endowment

Pure endowment benefits are dependent on the policyholder's survival at the policy's maturity date [11]. Pure endowment insurance benefits are not sold independently, but can be merged with term insurance benefits to produce the endowment insurance benefit.

The amount of benefit (b_{k+1}) of 1 unit is given after death if the insured is still alive for at least n- years since entering as an insurance participant, then:

$$b_{k+1} = \begin{cases} 0 \; for\; k \le n \\ 1 \; for\; k > n \end{cases}$$
$$v_{k+1} = v^n \; for\; k \ge n$$

The present value of a pure endowment benefit of $1 paid to a life aged x with a term of n years is *Z*, where:

$$Z = \begin{cases} 0 & for\; k = 0,1,2,\dots,n-1 \\ v^n & for\; k = n, n+1, \dots \end{cases} \quad (35)$$

The APV of the pure endowment benefit is denoted $A_{x:\overline{n|}}^{\;\;1}$ so that using the commutation symbol described in section 4, then:

$$A_{x:\overline{n|}}^{\;\;1} = v^n \;{}_n p_x$$

$$A_{x:\overline{n|}}^{\;\;1} = \frac{v^{x+n}\, l_{x+n}}{v^x l_x}$$

$$A_{x:\overline{n|}}^{\;\;1} = \frac{D_{x+n}}{D_x} \tag{36}$$

I. Endowment Insurance

Endowment insurance policy combines term insurance with a pure endowment [11]. If (x) dies within a fixed term, say *n* years, the sum insured is payable on death; however, if (x) survives for n years, the sum insured is payable at the end of the n[th] year. The amount of benefit (b_{k+1}) of 1 unit is given immediately after death or given shortly after the contract period expires and the insured is still alive, then

$$A_{x:\overline{n|}} = \begin{cases} v^{k+1} & for\ k \leq n-1 \\ v^n & for\ k > n \end{cases}$$

hence, the actuarial present value of endowment benefit for participants who *x* years old, with coverage period of *n* years indicated by $A_{x:\overline{n|}}$ and can be written as [4]:

$$\begin{aligned} A_{x:\overline{n|}} &= A_{x:\overline{n|}}^{1} + A_{x:\overline{n|}}^{\;\;1} \\ &= \left(\sum_{k=0}^{n-1} v^{k+1} ({}_k p_x)(q_{x+k}) \right) + (v^n)({}_n p_x) \end{aligned} \tag{37}$$

with ${}_k p_x$ is the probability that a person aged *x* year will survive to *k* years later, q_{x+k} is The probability that a person aged *x*+*k* years will die one year later, and ${}_n p_x$ is the probability that a person aged *x* year will survive *n* years later.

J. Prospective Reserves

Prospective reserves are reserves that are oriented to future expenditures or in other terms, namely the calculation of reserves based on the present value of all future expenses minus the present value of total future income for each policyholder [3]. With a benefit of IDR 1, the prospective reserves at the end of year *t* will be obtained as follows:

$${}_tV = (d_{x+k}.1.v + \cdots + d_{\omega-1}.1.v^{\omega-(x+k)}) - (l_{x+k}.P_x + l_{x+k+1}.P_x v + \cdots + l_{\omega-1}.P_x v^{\omega-(x+k)-1})$$

$${}_tV = 1.\left(d_{x+k}.v + \cdots + d_{\omega-1}.v^{w-(x+k)}\right) - P_x(l_{x+k}.v^0 + l_{x+k+1}.v + \cdots + l_{\omega-1}.v^{\omega-(x+k)-1})$$

$${}_tV = 1.\left(\frac{d_{x+k}}{l_x}.v + \cdots + \frac{d_{\omega-1}}{l_x}.v^{\omega-(x+k)}\right) - P_x\left(\frac{l_{x+k}}{l_x}.v^0 + \frac{l_{x+k+1}}{l_x}.v + \cdots + \frac{l_{\omega-1}}{l_x}.v^{\omega-(x+k)-1}\right)$$

$${}_tV = 1.\left({}_{t|}q_x.v + \cdots + {}_{\omega-1|}q_x.v^{\omega-(k+t)}\right) - P_x\left({}_k p_x.v^0 + {}_{k+1}p_x.v + \cdots + {}_{\omega-1}p_\omega.v^{\omega-(k+t)-1}\right)$$

$${}_tV = 1.\ A_{x+k} - P_x.\ \ddot{a}_{x+k} \tag{38}$$

mathematically, *t*-year-end prospective reserve formula for *n*-year endowment life insurance with 1 unit for someone aged x and y years indicated by ${}_tV_{xy:\overline{n|}}$ and can be written as follows:

$${}_tV_{xy:\overline{n|}} = A_{xy+t:\overline{n-t|}} - P_{xy:\overline{n|}}\ .\ \ddot{a}_{xy+t:\overline{n-t|}} \tag{39}$$

where $A_{xy+t:\overline{n-t|}}$ is compensation at the age of (*xy*+*t*) years and $P_{xy:\overline{n|}}.\ddot{a}_{xy+t:\overline{n-t|}}$ is cash value at age *(xy+t)* years remaining future premium.

K. Adjusted Reserve

An additional sources of funds to cover the cost of the beginning of the year can be obtained by adjusting premium reserve (adjusted reserve) [5]. These funds can be considered in the form of loans that will be paid later than gross premium payments in future years. Suppose P represents the amount of net premium for a certain type of insurance. The premium will be replaced by α in the first year and followed by β in the following years. α and β are adjusted premiums. The policyholder pays the same gross premium each year, i.e. P + costs. α and β only exist in actuarial calculations and have nothing to do with the policyholder.

Total cash value of $P = {}^{\alpha}\ Cash\ value\ +\ \beta\ Cash\ value$. This equation applies at the time the policy is issued. If n represents the period of reserve adjustment, then the relationship in the equation can be expressed mathematically as follows:

$$\alpha + \beta \ddot{a}_{x:\overline{n-1|}} = P\ddot{a}_{x:\overline{n|}} \tag{40}$$

Premium that is used to calculate the reserve value of the new jersey method is the continued net premium modified. $\alpha < P$, because part of P is used for the first year's costs, which is equal to $P - \alpha$. So, from the first year's net premium of P, there is only α provided to pay compensation in that year, the rest $P - \alpha$ is borrowed by the company and the loan will be paid later from the premium the following years. Because $\beta > P$, then $\alpha < P < \beta$ [5].

III. METHOD

The New Jersey method for calculating premium reserves for last survivor endowment life insurance starts with determining the insured's initial age when entering into the insurance contract, the interest rate (i), the amount of compensation to be received by the insured (b_{k+1}), the period of insurance coverage (n), and the mortality table used. The first step before calculating premium reserves is to calculate the cash value of premium payment using the commutation symbol. Second, compute the initial annuity value of the last survivor's term life insurance. After obtaining the initial annuity value, the next step is to calculate the value of the single annual premium of endowment for the last survivor. Next, calculate the annual net premium by substituting the initial annuity value and the annual premium. The final step is to calculate the value of the last survivor's endowment life insurance premium reserves.

IV. RESULT AND DISCUSSION

A. Last Survivor Insurance

Last Survivor Status is a status in which at least one member of a group of people survives and does not die. Last Survivor is represented by $(\overline{x_1, x_2, \ldots, x_m})$, where x_i indicates the age of member i of the insurance group and m denotes the number of members in the group. Considering the longest time-until failure distribution of the last survivor status, the number of members in the set is m people, which can be symbolized by $T(\overline{x_1}, \overline{x_2}, \ldots, \overline{x_m})$, it can be written as follows [4]:

$$T(\overline{x_1}, \overline{x_2}, \ldots, \overline{x_m}) = max[(T(x_1), T(x_2), \ldots, T(x_m)] \tag{41}$$

The probability of death in the case of two people aged x and y years during a period of n-years whose remaining life time is assumed to be mutually independent is expressed by [4]:

$$\begin{aligned} {}_nq_{\overline{xy}} &= P(T(\bar{x}, \bar{y}) \leq n) \\ &= P\,(\max[T(\bar{x}), T(\bar{y})] \leq n) \\ &= \mathrm{P}(T(x) \leq n, T(y) \leq n) \\ &= {}_nq_x \cdot {}_nq_y = 1 - {}_np_{\overline{xy}} \\ &= (1 - {}_np_x)(1 - {}_np_y) \end{aligned} \tag{42}$$

The probability that at least one person between x and y years will survive n years later is given by [4]:

$$\begin{aligned} {}_np_{\overline{xy}} &= 1 - {}_nq_{\overline{xy}} \\ &= 1 - (1 - {}_np_x)(1 - {}_np_y) \\ &= {}_np_x + {}_np_y - {}_np_x\,{}_np_y \\ &= {}_np_x + {}_np_y - {}_np_{xy} \end{aligned} \tag{43}$$

The probability that two insured persons aged x and y years will die in the interval between n and $n + 1$ year, is formulated as follows [4]:

$$\begin{aligned} {}_n|q_{\overline{xy}} &= P(n \leq T(\bar{x}, \bar{y}) \leq n + 1) \\ {}_n|q_{\overline{xy}} &= {}_nq_{\overline{xy}} - {}_{n+1}p_{\overline{xy}} \end{aligned} \tag{44}$$

Commutation Symbol for Last Survivor Insurance for the number of 2 insured persons aged x and y year are as follows:

$$D_{xy} = v^{\frac{x+y}{2}}.\,l_{xy} \tag{45}$$

$$N_{xy} = \sum\nolimits_{k=0}^{w} D_{x+k:y+k} = D_{x+0:y+0} + D_{x+1:y+1} + \cdots + D_{x+w,y+w}) \tag{46}$$

$$C_{xy} = v^{\frac{1}{2}(x+y)+1}.\, d_{xy} \tag{47}$$

$$M_{xy} = \sum_{k=0}^{w} C_{x+k:y+k} = C_{x+0:y+0} + C_{x+1:y+1} + \cdots + C_{x+w,y+w}) \tag{48}$$

The amount of the receipt of the last survivor annuity at the beginning of the period for n periods with an interest rate *i* each period is 1 rupiah for two people aged x and y is [4]:

$$\ddot{a}_{\overline{xy}:\overline{n|}} = 1 + v^1\,{}_1p_{\overline{xy}} + v^2\,{}_2p_{\overline{xy}} + \cdots + v^{n-1}\,{}_{n-1}p_{\overline{xy}}$$

$$\ddot{a}_{\overline{xy}:\overline{n|}} = \sum_{k=0}^{n-1} v^k\ {}_kp_{\overline{xy}}$$

$$\ddot{a}_{\overline{xy}:\overline{n|}} = \ddot{a}_{x:\overline{n|}} + \ddot{a}_{y:\overline{n|}} - \ddot{a}_{xy:\overline{n|}}$$

$$\ddot{a}_{\overline{xy}:\overline{n|}} = \frac{N_x - N_{x+n}}{D_x} + \frac{N_y - N_{y+n}}{D_y} - \frac{N_{x,y} - N_{x+n,y+n}}{D_{x,y}} \tag{49}$$

The single premium of last survivor endowment insurance for ages x and y, coverage period n years, and sum assured of 1 paid at the end of the policy year, can be stated as follows [3]:

$$A_{\overline{xy}:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1}.\, {}_{k|}q_{\overline{xy}} + v^n.\ {}_np_{\overline{xy}}$$

$$A_{\overline{xy}:\overline{n|}} = (A^1_{x:\overline{n|}} + A_{x:\overset{1}{\overline{n|}}}) + (A^1_{y:\overline{n|}} + A_{y:\overset{1}{\overline{n|}}}) - (A^1_{\overline{xy}:\overline{n|}} + A_{\overline{xy}:\overset{1}{\overline{n|}}})$$

$$A_{\overline{xy}:\overline{n|}} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x} + \frac{M_y - M_{y+n} + D_{y+n}}{D_y} - \frac{M_{x,y} - M_{x+n,y+n} + D_{x+n,y+n}}{D_{x,y}} \tag{50}$$

B. New Jersey Method

The New Jersey approach is a method of calculating the value of premium reserves by utilizing modified advanced net premiums. The *J* symbol denotes that the method is the New Jersey method. For example, if $P_{\overline{xy}:\overline{n|}}$ is the net premium for endowment last survivor insurance, then the premium will be replaced by α for the premium in the first year and followed by β in the following years, with a premium equal to adjusted are α and β. Where α^J and β^J are prospective premiums that have been modified using the new jersey method. The New Jersey method of calculating reserves gives a more effective value for insurance with premiums exceeding 20 payments. In the New Jersey method, the reserve value at the end of the first year is zero. The cash value of the premium in the first year is thus formulated mathematically as follows [5]:

$$\alpha^J = \frac{C_x}{D_x} \tag{51}$$

The amount of net premium is adjusted for the 2nd to *n*th year new jersey method denoted by β^J, which is obtained as follows:

$$\beta^J = P_{\overline{x:n|}} + \frac{P_{\overline{x:n|}} - \frac{C_x}{D_x}}{\ddot{a}_{\overline{x:19|}}} \tag{52}$$

The New Jersey method is part of the prospective reserve calculation. The value of prospective reserves in year *t* is the value of reserves based on the cash value of future benefits minus the cash value of future premiums. Calculating premium reserves adjusted to the New Jersey method at the end of the *t* year for insurance participants aged x years with a coverage period of n years indicated by ${}_tV^J{}_{\overline{x:n|}}$ and can be written as follows:

$${}_tV^J{}_{\overline{x:n|}} = b_{k+1}\ .\ A_{\overline{x+t:n-t|}} - \beta^J \ddot{a}_{\overline{x+t:20-t|}} - P_{\overline{x:n|}}(\ddot{a}_{\overline{x+t:n-t|}} - \ddot{a}_{\overline{x+t:20-t|}}) \tag{53}$$

with $A_{\overline{x+t:n-t|}}$ is net premium for a person aged of $(x + t)$ years with insurance period $(n- t)$ years, $P_{\overline{x:n|}}$ is annual net premium for a person aged x years with an insurance period of n years, and $\ddot{a}_{\overline{x+t:n-t|}}$ is initial annuity of a person aged of $(x + t)$ years with insurance period $(n - t)$ years.

The reserve value at the end of the first year is zero. So that the cash value of the premium in the first year for case two people aged *x* and *y* years can be written as following :

$$\alpha^J = \frac{C_{xy}}{D_{xy}}$$

$$\alpha^J = \frac{v^{\frac{1}{2}(x+y)+1}.\, d_{xy}}{v^{\frac{x+y}{2}}.\, l_{xy}} \tag{54}$$

$$\alpha^J = vq_{\overline{xy}}$$

β^J for participants life insurance last survivor two people who aged x and y years is

$$\beta^J = P_{\overline{xy}:\overline{n|}} + \frac{P_{\overline{xy}:\overline{n|}} - \alpha^J}{\ddot{a}_{\overline{xy}:\overline{19|}}} \tag{55}$$

then, New Jersey Method for Last Survivor endowment insurance as follows:

$$\begin{aligned} {}_tV^J_{\overline{xy}:\overline{n|}} &= b_{k+1}.A_{\overline{x+t,y+t}:\overline{n|}} - \beta^J \ddot{a}_{\overline{x+t,y+t}:\overline{20-t|}} - P_{\overline{xy}:\overline{n|}}(\ddot{a}_{\overline{x+t,y+t}:\overline{n-t|}} - \ddot{a}_{\overline{x+t,y+t}:\overline{20-t|}}) \\ &= b_{k+1}.A_{\overline{x+t,y+t}:\overline{n|}} - \left(\beta^J - P_{\overline{xy}:\overline{n|}}\right)\ddot{a}_{\overline{x+t,y+t}:\overline{20-t|}} \\ &\quad - P_{\overline{xy}:\overline{n|}}\, \ddot{a}_{\overline{x+t,y+t}:\overline{n-t|}}) \end{aligned} \tag{56}$$

C. Case Illustration

The calculation of premium reserves for policy holders aged 28 and 25 years with a contract period of n = 50 with payments made in t years until the contract period (n) with compensation of Rp. 100,000,000 using the New Jersey prospective method from the end of year 1 to the end of year 50 are always increasing at the beginning of the insurance contract and approaches compensation at the end of the period. This demonstrates that many companies have failed because the calculation of the annual premium reserve before the coverage period expires does not reach the reserve fund. The results of this calculation can be used by insurance firms to create claims fund agreements so that the company can make compensation before the coverage period has ended and avoid losses. Total premium payments for reserves using the New Jersey method for male (28) and Female (25) with a 50-year insurance contract can be seen in Figure 1.

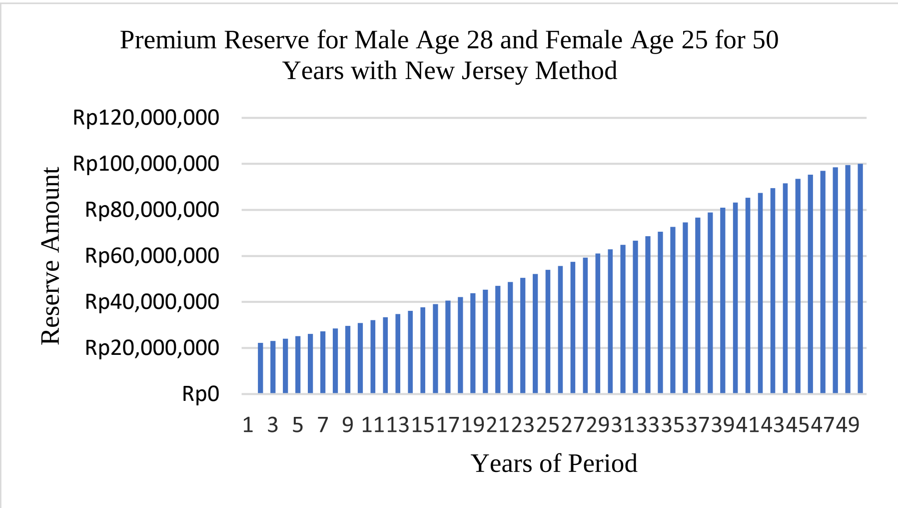

Figure. 1 Estimation of Premium Reserve Policyholder age 28 with 25 years old

in Figure 1 it can be seen that the value of the reserves at the end of the first year using the New Jersey prospective method ($t = 1$) is Rp.0.00, so the reserves in the New Jersey method start in year 2, for t years with $t = 2, 3, \ldots, n$ where n is the contract period of the insurance participant ($n = 50$). At the end of the premium payment period, the new jersey reserve generates a value that corresponds to the compensation, which is Rp. 100,000,000. It can be concluded that the calculation of premium reserves using the New Jersey method is determined by the length of the insurance contract and the age of the insured at the time of insurance.

IV. CONCLUSION

According to the results of the simulation, the value of premium reserves with the age of male x = 28 and female y = 25 and a contract period of n = 50 with payments made in t years until the contract period ($n = 50$) with compensation of Rp. 100,000,000 using the New Jersey prospective method from the end of year 1 to the end of year 50 is always rising. The value of reserves is determined based on the initial age of a person when starting insurance, the older the initial age of the insurance participant, the greater the amount of the reserves obtained by the company. This is due to the higher death rate in the elderly age.

Based on the discussion, it can be concluded that using the New Jersey method to determine the value of the annual premium reserve for endowment life insurance in the last survivor case is more effective because the premium paid by insurance participants in the first year does not need to be reserved by the insurance company, so the insurance company can use it for their needs.

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