

Portfolio Optimization Analysis Using Markowitz Model on Idx30 Stock Index in 2022 and 2023

Edwin Setiawan Nugraha^{a*}, Carlina Juliany Lantang^a, Mokhammad Ridwan Yudhanegara^b

^aStudy Program of Actuarial Science, Faculty of Business, President University, 17550 Indonesia

^cDepartment of Mathematics Education, Universitas Singaperbangsa Karawang

*edwin.nugraha@president.ac.id

Abstract –Economic growth today has an impact on the lives of residents of a country. Investing in the stock market involves a relatively high degree of risk, as stock prices can fluctuate very quickly. However, a proper analysis in forming a portfolio is very important before making any investment decision to get maximal return. This research will use Markowitz Model to get optimal. This model explains the importance of diversification and how it can reduce overall risk while increasing returns. Data to be use are from IDX30 Index Companies in 2022 and 2023 because this index includes stocks from various industrial sectors that reflects significant developments in the Indonesian stock market. Data processing in this research will use Python. This model will determine best combination to become optimal portfolio from list companies in IDX30 Index. The result from this research shows that there 7 stocks become the combination for the optimal portfolio with the expected returns rate of is 65.776% and the portfolio risk is 20.0008%. The weights for each stock in optimal portfolio obtained using PyPortfolioOpt package in Python, are BBNI.JK = 12.024%, BMRI.JK = 16.142%, INCO.JK= 5.033%, INDF.JK = 12.36%, ITMG.JK = 38.606%, KLBK.JK = 11.148%, UNVR.JK.=4.688%. Markowitz model can investor investor an consideration to select the stocks recommendation with the optimal weight for their portfolio to get maximum return with low risk.

Keywords: Type your keywords or phrases here, 3-5 keywords, each separated by a comma

Introduction

The development of stocks and investment in Indonesia continues to increase along with the development of the Indonesian economy which is growing. From Figure 1, it shows that start from 2000s the Gross Domestic Product (GDP) in Indonesia increase until 2020. Since the introduction of stock trading systems, the number of investors in the Indonesian stock market has been steadily increasing, with retail investors in particular increasingly enjoying the benefits of investing in the stock markets. Moreover, the Indonesian government seeks to further support the development of the stock market through various policies and regulations aimed at protecting and welfare of investors. Considering these increasingly positive developments, it is expected that the stock market and investment in Indonesia will keep grow and has a positive impact on Indonesia's economic growth

An investment purchase is the use of money or assets with the expectation of future profits or returns. Investing takes many forms, such as property, currency, stocks, bonds, and etc. One of the most frequently used forms of investment is stocks. Stock is the unit of ownership of a company. Stocks are traded between investors on the stock exchange. In Indonesia, there is Indonesia Stock Exchange (IDX) as stock market in Indonesia, which is the place to buy and sell stocks and also regulate them. Stock prices can rise or fall depending on company performance and market sentiment. If the company grows and is in good financial standing, the stock price will rise and vice versa.

Investing in the stock market involves a relatively high degree of risk, as stock prices can fluctuate very quickly and violently. However, the higher the risk, then higher the chance

of high return. Therefore, the important thing before investing in the stock market is investors need to understand the risks involved in investing and conduct a proper analysis before making any investment decision to get maximal return. One way to get maximum returns is to build a portfolio.

Portfolio optimization using the Markowitz method is a popular technique in investment management. This technique was introduced by Markowitz (1952) and suggested that investors could optimize their portfolios by diversifying their investments across different asset classes with different returns. This method analyzes the expected return and risk associated with each asset in the portfolio and allocates the assets to maximize the expected return for the portfolio in a given level of risk.

Problems arise when investors face challenges in constructing efficient portfolios using the Markowitz method. One of the main problems is the difficulty in predicting the accuracy of inputs such as expected returns and risks associated with different assets. Also, this method assumes that all investors have similar risk and return preferences, which may not be true. Furthermore, this methodology does not take into account transaction costs, taxes and liquidity constraints that may affect portfolio allocation.

Portfolio optimization using Markowitz theory is very important as investors usually face the challenge of choosing the right assets to achieve their investment objectives. The big risk in investment the stocks is when investors just invested in only one stock. When it comes to securities, high-yield investments tend to carry more risk, so low-risk investments tend to offer lower returns. In this case, the Markowitz model helps investors identify and mitigate risks associated with their investment portfolios.

Portfolio optimization using Markowitz theory provides investors with risk mitigation benefits. This is achieved by collecting and analyzing financial market data and building efficient and optimal portfolios. Therefore, this theory becomes very important for investors to make sound and effective investment plans. This theory will produce a combination of stocks to buy that will be used in the portfolio that produces maximum returns with low risk.

There are several methods that can use to get optimal portfolio, such as Markowitz model, Maximum Diversification Portfolio, Conditional Value at Risk (CVaR) Optimization, Short-selling Constraints Optimization, Mean Absolute Deviation (MAD) Portfolio Optimization, and more. This research will focus discuss about using Markowitz model to get optimal portfolio. Author chooses this method because Markowitz model focuses on portfolio that considers the risk and return at the same time, so that it does not only focus on the return to be obtained and allowing you to create the most efficient portfolios. This model considers risk within a portfolio by considering different types of assets and how they correlate with each other (Markowitz,1952).

In literature, there are previous studies discuss about optimal portfolio using Markowitz model and other model. For example, Abdurrazak (2017) implement Markowitz model to choose optimal portfolio using data from Jakarta Composite Index. Next from Dewi (2019) discuss about comparison result of optimization portfolio between Markowitz model and Single Index Model using data from LQ45 Index in 2017 until 2018. There are more studies discuss about optimization portfolio using Markowitz or using different models, however most of them processing the data using Microsoft excel (Pracanda et al.,2017; Anam, 2021; Rusmiati et al., 2022; Hakim & Waluyo, 2023). In this work, the author implement Python software on

processing the data in this research and the data used from IDX30 Index.

Stock Price

Stocks are formed when a company needs capital for the development of its company. To overcome this, the company sells part of the company's ownership in the stocks with an IPO or Initial Public Offering (Inrawan, 2022). The sale of these stocks is open to the public, but there are rules governing made by the Indonesia Stock Exchange (IDX). Indonesia Stock Exchange (IDX) is in charge of carrying out and managing the regulation of buying and selling stocks. Every activity regarding buying and selling of stocks will be recorded in the IDX. However, the investors must be able to consider the capability of the company. This is because the selling price of the stocks will follow the development of the company.

The stock price will determine the profit and loss for the investors. The company will determine the stock price according to the condition or performance of the company. The performance of the company can be assessed from the financial reports. Companies that have sold stocks to the public must publish their financial statements to investors. If the financial statements show an increase, the investor's profit will increase due to rising stock prices. Otherwise, if there is a decrease, then the stock price will also fall.

Investors who own stocks in a company expect a return to turn a benefit to them. According to (Inrawan, 2022), return is the expected return on investment by the company or investors. The expectation of a return occurs in financial assets, demonstrating investor ability to invest their capital to gain benefits over time as long as the funds are invested and the risks that are obtained. Certainly, to invest and obtain a return, an investor must focus on the risks that can occur. A risk is the difference between the potential value of the actual return and the expected return (Tandelilin, 2010).

Risk

Investing in stocks is one way to generate financial returns in the stock market. However, like any other investment, stock trading also comes with risks that need to be well understood before deciding to invest in it. According to (Tandelilin, 2010), risk is the possible difference that occurs between the actual return and the expected return. One approach that can be adopted to minimize risk is by implementing portfolio diversification. Diversification is an investment strategy that invests in a variety of stocks or other financial instruments. By diversifying, an investor can reduce the risk associated with a particular stock or sector. If one of the stocks in your portfolio falls in price, the others may not be affected or even increase in price. However, despite the diversification, risk still exists in the world of stocks.

Investors will choose stock for their portfolio with minimize the risk for and get the optimal portfolio. This research will use Markowitz model to get the optimal portfolio and this model consider risk in build the optimal portfolio. There are several factors that will affect the level of risk, such as the covariance, variance, standard deviation, and coefficient correlation. The risk will depend on the value from that.

IDX30

IDX30 is an index that measures the performance from the 30 stocks that have high

liquidity and large market capitalization and are supported by good company fundamentals (IDX, 2023a). This stock index certainly makes it easier for investors to narrow down their choices when they want to invest in stocks that are liquid and have a large market capitalization. This index is managed by third parties outside the IDX, but IDX continues to carry out minor and major evaluations of the IDX30 stock index group every 3 and 6 months.

General information regarding the selection of components of the IDX30 indicator is as follows (IDX, 2023b). The first one is IDX30 index constituents are selected from stocks included in the calculation of the LQ45 index in the same period. The second one is the factors considered in selecting the constituents of the IDX30 index are transaction activities such as the value from transaction, frequency of transaction, number of transaction days in the regular market and free float market capitalization. The third one is apart from using quantitative criteria, the IDX also considers financial conditions, growth prospects and other factors related to the sustainability of a company's business. IDX conducts a review of the IDX30 index every 6 months to take effect in early February and August.

Portfolio

Portfolio investment is the allocation of funds among various financial assets such as stocks, bonds, and mutual funds in order to achieve a diversified investment portfolio for the investor (Hartono, 2020). A series of securities or assets that are bought and sold with the goal of generating a return on investment. Because securities perform differently in different economic conditions, diversifying securities within a portfolio investment reduces risk and optimizes returns.

However, portfolio investing can come with risks and challenges. Market volatility can affect portfolio returns, and investors may also face issues with market timing and forecasting. Additionally, international portfolio investments can come with regulatory and regulatory challenges. Therefore, investors should do thorough research and find the advice of a financial advisor before investing in any portfolio.

Markowitz Model

Investors will buy stocks that will become a portfolio. This portfolio presents risk and return for investors. Risk and return are related and cannot be separated because the higher the risk, the more return the investor will get. Otherwise, if the return is low, the return received by the investor will be low. Investors need to control and choose which stocks to buy in order to maximize their profits. There are several ways to construct an optimal portfolio, one of which is the Markowitz model.

Markowitz Theory, also known as Modern Portfolio Theory (MPT), is a mathematical framework for portfolio optimization developed by Markowitz (1952). This theory emphasizes the importance of diversification and how it can reduce overall risk while increasing returns. Markowitz theory has had a major impact on the fields of finance and portfolio management and is still widely studied and applied today.

The basic theory of Markowitz theory is that investors should not only focus on maximizing return, but also minimizing risk. Markowitz argued that investors should focus on building portfolios that offer the best return for a level of risk that already defined, or portfolios

that offer the lowest level of risk for a given level of return. This principle is known as the efficiency frontier. This is the set of all portfolios that offer the highest expected return for a level of risk that already defined.

Markowitz model focused on relation between risk and return investment. This model can handle random diversification, which is the formation of a portfolio with selecting securities randomly without regard to the characteristics of the relevant investments (Suprianto, 2008). The Markowitz model uses statistical analysis to analyze the returns and risks for the various assets and determine the optimal allocation of assets within a portfolio. This model assumes that investors act rationally and seek to maximize returns while minimizing risk.

Markowitz also introduced the concept of diversification, the idea of spreading an investment across multiple assets to reduce overall risk. This means that an investor should not invest all his money in one asset, but should invest in a mix of stocks, bonds, and other instruments to reduce the risk of the entire portfolio

Research Method

This research will use list data companies that listed on the IDX30 Stock Index within January 1st, 2022 until January 31st, 2023. The data is taken from Indonesia Stock Exchange (IDX) and there are 33 companies that listed on the IDX30 Stock Index within the period (IDX 2023c). The list of companies is written in the table below.

Table 1. Companies listed in IDX30 index within 2022 – 2023

No	Code	Company
1	ADRO	PT. Adaro Energy Indonesia Tbk.
2	AMRT	PT. Sumber Alfaria Trijaya Tbk.
3	ANTM	PT. Aneka Tambang Tbk.
4	ARTO	PT. Bank Jago Tbk.
5	ASII	PT. Astra International Tbk.
6	BBCA	PT. Bank Central Asia Tbk.
7	BBRI	PT. Bank Rakyat Indonesia Tbk.
8	BBNI	PT. Bank Negara Indonesia Tbk.
9	BMRI	PT. Bank Mandiri Tbk.
10	BRPT	PT. Barito Pacific Tbk.
11	BUKA	PT. Bukalapak.com Tbk.
12	CPIN	PT. Charoen Pokphand Indonesia Tbk.
13	EMTK	PT. Elang Mahkota Teknologi Tbk.
14	ESSA	PT. Surya Essa Perkasa Tbk.
15	GOTO	PT. Goto Gojek Tokopedia Tbk.
16	HRUM	PT. Harum Energy Tbk.
17	ICBP	PT. Indofood CBP Sukses Makmur Tbk.
18	INCO	PT. Vale Indonesia Tbk.
19	INDF	PT. Indofood Sukses Makmur Tbk.
20	INKP	PT. Indah Kiat Pulp & Paper Tbk.
21	ITMG	PT. Indo Tambang Raya Megah Tbk.

22	KLBF	PT. Kalbe Farma Tbk.
23	MDKA	PT. Merdeka Copper Gold Tbk.
24	MEDC	PT. Medco Energy International Tbk.
25	PGAS	PT. Perusahaan Gas Negara Tbk.
26	PTBA	PT. Bukit Asam Tbk.
27	SMGR	PT. Semen Indonesia Tbk.
28	TBIG	PT. Tower Bersama Infrastructure Tbk.
29	TINS	PT. Timah Tbk.
30	TLKM	PT. Telkom Indonesia Tbk.
31	TOWR	PT. Sarana Menara Nusantara Tbk.
32	UNVR	PT. Unilever Indonesia Tbk.

The characteristics that will be selected on research is purpose sampling method for this research is company listed consistently in the IDX30 Stock Index in last 2 periods (2022-2023). According to this sampling, there are 27 company fulfilled this characteristic.

By follow Hartono (2020), the determination of the optimal portfolio using the model Markowitz can be done with several steps, namely:

1. Calculate Return from each Stock using equation

$$R_{it} = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where, R_{it} is rate of return actual, P_t is stock price at time t , P_{t-1} Stock price at time $t - 1$

2. Calculate Expected Return each stock using equation

$$E(R_i) = \frac{1}{N} \sum_{t=1}^N R_{it}$$

where, R_{it} is rate of return actual, $P_t =$ Stock price at time t , $t-1 =$ Stock price at time $t - 1$, and N is number observation.

3. Calculate Risk which Variance and Standard Deviation from each stock. This measure of spread is meant to find about how far it is possible for the value to get deviates from that value expected. Variance formula is given by

$$\sigma_i^2 = \frac{\sum_{j=1}^N (R_{ij} - E(R_i))^2}{N}$$

and standard deviation is $\sigma_i = \sqrt{\sigma_i^2}$

4. Calculate the covariance of 2 stocks in a portfolio using the following equation

$$Cov(R_a, R_b) = \frac{1}{N} \sum_{i=1}^N (R_{ai} - E(R_{ai}))(R_{bi} - E(R_{bi}))$$

where R_a is stock return a in t period, R_b is stock return b in t period, $E(R_a)$ is expected return of stock a, $E(R_b)$ is expected return of stock b and N is number of observations.

5. Calculate weight for each stock by using this equation. In this step, calculation of weight using PyPortfolioOpt package in python.
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6. Calculate Expected Return Portfolio using equation

$$E(R_p) = \sum_{i=1}^n W_i E(R_i)$$

where $E(R_i)$ is expected return of stock I, W_i is weight for stock i, n is number stock.

7. Calculate Portfolio Risk using equation (Hartono, 2020):

$$\sigma_p = \sqrt{\sum_{i=1}^n \sum_{j=1}^n W_i W_j \sigma_{ij}}$$

where σ_p is standard deviation of portfolio, W_i is weight of stock i, W_j is weight of stock j and σ_{ij} is covariance between stock i and stock j, n is number of stock.

Results and Discussion

Calculate Expected Return, and Risk

Stocks in candidate of the optimal portfolio are known from the expected return for each stock. Expected return is calculated by dividing the realized return rate by the research period. The result of calculation is shown at Table 3. Stocks that will use in this research are who have the positive expected returns. Negative expected returns means that investor will suffer a loss if they make investment on that stock. Stocks that have negative expected returns will be remove from the list candidate.

Table 3. Expected returns, standard deviation and variance
Each Stock

No.	Code	Expected Return	Standard Deviation	Variance
1.	ADRO	0.001736	0.028808	0.000827
2.	ANTM	0.000428	0.030366	0.000919
3.	ASII	0.000403	0.047167	0.002216
4.	BBCA	0.000827	0.017736	0.000313
5.	BBNI	0.001450	0.014519	0.000210
6.	BBRI	0.000688	0.017787	0.000315
7.	BMRI	0.001631	0.015864	0.000251
8.	BRPT	0.000304	0.017647	0.000310
9.	CPIN	0.000208	0.024509	0.000598
10.	INCO	0.002073	0.037848	0.001427
11.	INDF	0.000554	0.019017	0.000360
12.	ITMG	0.003419	0.033587	0.001124
13.	KLBF	0.001313	0.046193	0.002123
14.	MDKA	0.000916	0.033782	0.001137
15.	PGAS	0.000948	0.030070	0.000901
16.	PTBA	0.001842	0.012699	0.000161
17.	SMGR	0.000304	0.026068	0.000677
18.	TLKM	0.000056	0.018058	0.000325
19.	TOWR	0.000201	0.030620	0.000934

20.	UNTR	0.000994	0.022625	0.000510
21.	UNVR	0.000799	0.023237	0.000538

Risk describes the possibility of a deviation from realized returns and expected returns. Return and risk tend to move together, which means that high-return stocks also tend to be high-risk. Calculate risk can use variance and standard deviation from each stock that listed on above table. Define candidate for optimal portfolio using comparison between expected returns and the risk from each stock. The value that will represent the risk in comparison with expected returns is variance. In calculating the risk of each stock, the value of variance will represent the risk because in Markowitz model variance give information about how far an actual may differ from the expectations. Besides that, variance will affect in decide the portfolio investment. If the expected returns from the stock bigger than the variance from the stock, then the stock will become the candidate for optimal portfolio and will be used in the next steps. The comparison from expected returns and variance shown in the Table 4.

Table 4. Stocks that have Expected Returns greater than Variance

No.	Code	Expected Returns	Comparison	Variance
1.	ADRO	0.001736	>	0.000827
2.	ASII	0.000403	>	0.002216
3.	BBCA	0.000827	>	0.000313
4.	BBNI	0.001450	>	0.000210
5.	BBRI	0.000688	>	0.000315
6.	BMRI	0.001631	>	0.000251
7.	INCO	0.002073	>	0.001427
8.	INDF	0.000554	>	0.000360
9.	ITMG	0.003419	>	0.001124
10.	KLBF	0.001313	>	0.002123
11.	PGAS	0.000948	>	0.000901
12.	PTBA	0.001842	>	0.000161
13.	UNTR	0.000994	>	0.000510
14.	UNVR	0.000799	>	0.000538

Covariance 2 Stocks

Covariance is a measure the direction of movement from 2 variables. A positive covariance means that the combination of two stocks in an exclusive portfolio tends to move in the same direction. A negative covariance shows that two stocks move in opposite directions, in one direction, if one stock increases in profit the other will decrease in return. The result from calculation the covariance from the candidate optimal portfolio on the Table 5.

Table 5. Covarian between stocks

	S1	S2	S3	S4	S5	S6	S7	S8	S8	S10	S11	S12	S13	S14
S1	8.30	1.23	0.35	0.60	0.99	1.10	2.60	-0.13	0.500	0.29	2.50	4.54	3.56	-0.24
S1	1.23	3.15	0.57	0.83	0.65	0.95	1.04	0.20	0.37	0.36	0.76	0.56	0.79	0.24
S3	0.35	0.57	211	138	115	116	002	033	018	067	044	032	052	048
S4	0.60	0.83	138	316	161	148	047	010	041	057	074	057	043	067
S5	0.99	0.65	115	161	252	135	049	014	022	050	069	062	055	037
S6	1.10	0.95	116	148	135	311	049	016	068	062	069	062	081	034
S7	2.60	1.04	002	047	049	049	904	-053	173	042	131	161	096	-060
S8	-0.13	0.20	033	010	014	016	-053	161	-011	062	008	010	-011	060
S9	5.00	0.37	018	041	022	068	173	-011	680	032	245	406	296	-040
S10	0.29	0.36	067	057	050	062	042	062	032	326	045	040	050	076
S11	2.50	0.76	044	074	069	069	131	008	245	045	512	241	170	-008
S12	4.54	0.56	032	057	062	062	161	010	406	040	241	540	256	-054
S13	3.56	0.79	052	043	055	081	096	-0011	296	050	170	256	469	017
S14	-0.24	0.24	048	067	037	034	-060	060	-040	076	-008	-054	017	476

* x 10⁴

Portfolio Optimization

The optimal portfolio is on the efficient set line, or in other hand optimal portfolio is part of the efficient portfolio. Optimal portfolio can get with define the level of risk, then maximizethe expected returns for the portfolio. In this research, calculation of efficient frontier or efficient set will calculate in Python with find the minimum variance portfolio, also known asMVP. The result from calculation the MVP using Python is 0.0001.

In addition to calculating the minimum variance portfolio (MVP), it is necessary to calculate the expected return portfolio that will become the y-axis in efficient frontier. To calculate the expected return portfolio, need the value of the weights or the optimal weights. In this research the weights will not pre-define, but will use calculation in Python. Calculationsneeds because in Markowitz model make diversification in the portfolio. The important thingin calculate the weights for the portfolio is the total of the weights from all stock have to equal1 that means the funds allocated 100%. The result of the calculation using Python are presented at Table 6.

Table 6. Stocks for The Optimal Portfolio

No.	Code	Weights
1.	BBNI.JK	12.024%,
2.	BMRI.JK	16.142%,
3.	INCO.JK	5.033%
4.	INDF.JK	12.36%
5.	ITMG.JK	38.606%
6.	KLBF.JK	11.148%

7.	UNVR.JK	4.688%
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Conclusion and Recommendation

Conclusion

This research use list companies in IDX30 Stock Index that listed during Jan 2022 until Jan 2023. There are 33 stocks as the population in this research and there are 27 stocks become the sample for this research. Our analysis shows that there are 7 stocks that become the combination for the optimal portfolio in this research. They are BBNI.JK, BMRI.JK, INCO.JK, INDF.JK, ITMG.JK, KLBF.JK, UNVR.JK with weight 12.04%, 16.142%, 5.033%, 12.360%, 38.606%, 11.148%, and 4.688%, respectively. The expected returns from the optimal portfolio are 0.211% and the portfolio risk is 0.011%. This portfolio can maximize the return for investor and investor have low risk for the portfolio, because the expected returns from the portfolio bigger than the risk of the portfolio.

Recommendation

The result of this research give recommendation for investor to do the diversification when build their portfolio because the result from this research proves that with diversification the risk of the portfolio will decreased.

There is no guarantee that the portfolio results will match reality because stock prices are determined by many factors, for example political factors, security factors, food availability factors, inflation and other factors. We think that investors need to integrate a portfolio of the results of this research and other factors that influence stock prices in making an investment strategy or decision.

There are more model or method that can use to get the optimal portfolio. In the next research can try the other method to get the optimal portfolio and compare with this method because the combination for optimal portfolio in this research not necessarily become the optimal portfolio when using other models.

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