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Construction Cost Reduction in Design of Prestressed Concrete Structural System

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Abstract

Concrete material has moderately strong compressive strength, but relatively weak tensile strength. To overcome this problem, three kinds of systems can be applied, namely reinforced concrete system, composite concrete system, and prestressed concrete system. In a prestressed concrete system, a compressive force is applied to annihilate the tension region in the concrete section. Compared to reinforced concrete, prestressed concrete requires a smaller section of concrete, since the whole cross section is in compression and active. However, prestressing concrete needs the use of high strength steel wire which is extremely costly. The use of prestressing concrete may be carried out using minimization on the use of such expensive high strength steel. In this research, two methods of minimization of the use of high strength steel, which are the shifting of the support of the beam to get the smaller field moment. The other method is to reshape the concrete section to have a higher moment of inertia. It is found out that the two methods perform well in the minimization process of construction cost of the prestressing concrete beam.

Keywords: prestressed concrete, construction cost reduction, bridge structure, support shifting, section reshape

1. Introduction

Concrete is a mixture of sand, gravel, crushed stone, or other aggregates mixed with a paste of cement and water to form a rock-like mass. One or more additives may be added to create concrete with specific properties such as workability, durability, and cure time. Like other rock-like materials, concrete has high compressive strength and very low tensile strength. Prestressed concrete is a combination of concrete and steel, where the steel provides tensile strength not found in ordinary concrete [1-3]. To overcome the weakness of concrete in tension, one may use one of the following methods. The first method is to use mild steel in the region of tension, so that the tensile region is replaced by then tension in steel bars. This method is called reinforced concrete. In this method, the steel bars play their role after the external loads cause tension in the section of the component. So, in this method, tensile region still exists. To guaranty that the failure of the component is the ductile tension failure, the percentage of used steel bar is limited, for example, 2 % for bending components and up to 6 % for vertical components.

The component that requires more steel reinforcements may be made of composite concrete. In this method, instead of steel bars, steel profile is embedded in the concrete to overcome the tension region. The tension still exists, but higher percentage of steel is allowed to be used. Another method is the use of prestressed concrete [4-7]. A strand is tensioned to a certain degree of prestress, and then the concrete is poured, After the concrete gains enough compression strength, the strand two ends are cut. The steel tends to return to its original shape, and this tendency creates compression in the concrete, such that the tensile region due to the external loadings is eliminated [8-10]. The application of prestressing concrete needs more

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sophisticated equipment and more costly materials. So, optimization process in the design scheme is preferable [11-12]. To get the optimal design, several approaches in minimization or the optimization of the design of concrete prestressing system can be done, namely the shifting of the supports toward the interior of the beam and the enlargement of the moment of inertia of the section. These two approaches are proposed in this study to minimize the construction cost of the prestressing concrete beam.

2. Optimization of Prestressed Concrete System

The optimization may be carried out in the selection of the materials used, the design method and the selection of structural geometry. One method of optimization procedure in the design of the prestressing concrete, is the minimization of the span of the structure to make the actual bending moment minimum. In the design of a concrete beam for example, the beam initially constructed as a simple beam, the moment may be reduced if the supports are moved inside the span. By moving the support inside, it may obtain beam overhang that may reduce the bending moments. This way of optimization process will be investigated throughout the discussion of this research. Consider a bridge structure made of simple beams, shown in Fig. 1. The maximum moments due to the external load q are directly proportional to the square of the span L and uniformly distributed load q, yields the following equation.

$$M = \frac{1}{8} q L^2 \tag{1}$$

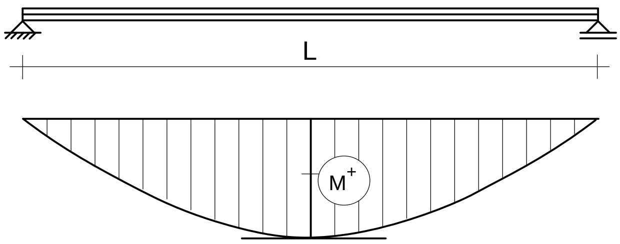

Fig. 1 Simple beam without pedestal shift

and the shear on the support becomes

$$V = \frac{1}{2} q\, L \tag{2}$$

In the design with the shifting of the supports, both supports are moved at a distance s inward, so the maximum negative moment at support

$$M^- = -\frac{1}{2} q s^2 \tag{3}$$

while the maximum positive bending moment at mid span is

$$M^+ = +\frac{1}{8} q (L - 2s)^2 - \frac{1}{2} q s^2 \tag{4}$$

To minimize the maximum positive moment in the middle of the span, both positions are shifted as far as a equal to the inside, such as that the absolute value of the negative moment above the support, is equal to the absolute value of the positive moment in the middle of the span. See Fig. 2 for explanation.

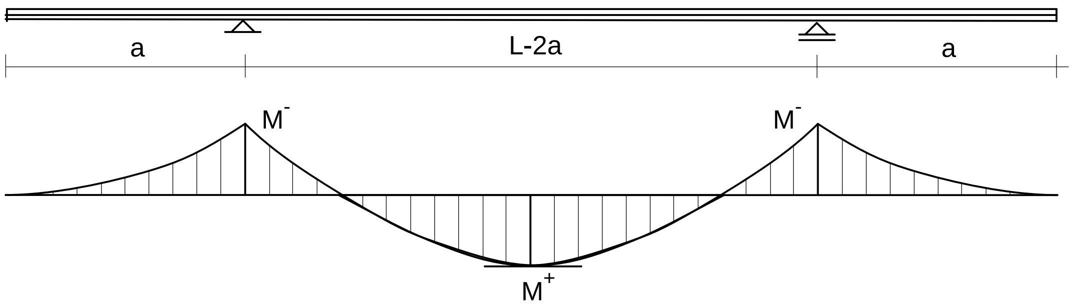

Fig. 2 Simple beam with pedestal shift

An optimum design is obtained by adjusting the value of s such that the absolute value of the negative moment is the same with the positive moment,

$$+\frac{1}{8}q(L-2s)^2-\frac{1}{2}qs^2=+\frac{1}{2}qs^2 \tag{5}$$

which yields

$$s=0.20711\,L \tag{6}$$

and therefore

$$M^{+}=-M^{-}=+0.02145\,qL^2 \tag{7}$$

which is only about 17 % of the maximum moment before the shifting of the supports inside in equation (1).

3. Application of Prestressed Concrete Design Optimization

The minimalization or the optimization of the design of concrete prestressing system can be implemented by several approaches. One of the methods, is the shifting of the supports toward the interior of the beam. The other is the enlargement of the moment of inertia of the section or the reshape of the beam section. Examples of these two methods will be discussed in this section. At the end of this section, the comparison between the unoptimized and the optimized design results is also discussed.

3.1 Design of prestressed concrete beam without optimization

In this example the design of simply supported concrete prestressing beam uses the following materials, geometry and external forces [13-14].

(1) Material
- (a) Concrete:
  - ❑ Characteristic compressive strength, $f_c'=50$ MPa
  - ❑ Allowable compressive stress,
    - $\bar{f}_{cs}=0.45\,f_c'$ = 22.50 MPa (while serving)
    - $\bar{f}_{ct}=0.60\,f_c'$ = 30.00 MPa (during transfer)
  - ❑ Allowable tensile stress,
    - $\bar{f}_{ts}=-\,0.50\,\sqrt{f_c'}$ = - 3.535 MPa (during transfer)
    - $\bar{f}_{tt}=-\,0.25\,\sqrt{f_c'}$ = - 1.768 MPa (while serving)
  - ❑ Elastic modulus: E_s = 200 000 MPa
- (b) Prestressed steel:
  - ❑ Yield strength : $f_{py}=1680$ MPa
  - ❑ Tensile strength : $f_{pu}=1860$ MPa

- Elastic modulus : E_s = 200 000 MPa
- Allowable stresses : 0.82 f_{py} or 0.74 f_{pu}

(c) Ordinary reinforcing steel:

- Yield strength: $f_y = 400$ MPa (longitudinal reinforcement)
  $f_y = 350$ MPa (stirrup)
- Elastic modulus: E_s = 200 000 MPa

(2) Structural geometry

- Total span : L = 15.00 m
- Cross section u : A_c = b x d =0.40 m x 0.75 m = 0.30 m^2
- Moment of inertia : I = $\frac{1}{12}bd^3 = 0.01416$ m^4
- Concrete cover : s = 4.0 cm (horizontal), 8.0 cm (vertical)
- Girder space : S_g = 1.25 m
- Stirrup space : s_v
- Thickness of slab : t_s = 0.200 m
- Thickness of pavement : $t_p = 0.075$ m
- Area of tendon : A_p
- Stirrup width : A_v

(3) External force

- Girder's self-weight : $q_g = 0.4 \text{ x } 0.75 \text{ x } 25.00 = 7.50 \text{ kN/m}$
- Superimpose dead load : $q_s = (0.20 \text{ x } 25.00 + 0.075 \text{ x } 22) \text{ x } 1.25)$
- Uniformly distributed live load : $q_\ell = 9\left(0.50 + \frac{15}{30}\right) \text{x} 1.25 = 11.25 \frac{\text{kN}}{\text{m}}$
- Line live load : $P = 49 \text{ x } 1.25 = 61.25 \text{ kN}$

(4) Internal force

Mid span moment:

- Moment of self-weight girder : $M_g = \frac{1}{8}q_gL^2 = 210.94$ kNm
- Dead superimpose : $M_s = \frac{1}{8}q_sL^2 = 233.72$ kNm
- Live uniform load moment : $M_\ell = \frac{1}{8}q_\ell L^2 = 316.41$ kNm
- Live point load moment : $M_p = \frac{1}{4}P\ L = 229.69$ kNm
- Total moment : M_t = 990.76 kNm
- Dead load moment : M_d = 444.66 kNm

Pedestal shear forces:

- Shear of the girder's self-weight : $V_g = \frac{1}{2}q_gL = 56.25$ kN
- Dead superimpose : $V_s = \frac{1}{2}q_sL = 62.33$ kN
- Live uniform load moment : $V_\ell = \frac{1}{2}q_\ell L = 84.38$ kN
- Live point load moment : $V_p = \frac{1}{2}P = 30.63$ kN

Loads, moments, shear for all loading cases are given in Table 1.

Table 1 Loads, moments and shear forces for no optimization case

Case	Loading	*M* (kN-m)	*V* (kN)
Beam weight	7.500 kN/m	210.94	56.25
Superimpose	8.313 kN/m	233.72	62.33
Uniform live	11.250 kN/m	316.41	84.38
Point live	61.250 kN	229.69	30.63
Total	-	990.76	

(5) Prestressed force and mid-span eccentricity

To calculate the prestressed force and eccentricity of the cable in the center of the span, a Magnel diagram is used. The giration radius is

$$r_{yy}^2 = \frac{I_{yy}}{A} = \frac{1}{12}d^2$$

and the boundary of the Kern area is

$$k_t = -\frac{r_{yy}^2}{y_b} = +\frac{1}{6}d = +0.125 \text{ m}: k_b = -\frac{r_{yy}^2}{y_t} = -\frac{1}{6}d = -0.125 \text{ m}$$

and

$$\alpha_t = \frac{A\, y_t}{I_{yy}} = +8.000 \text{ m}^{-1}; \ \alpha_b = \frac{A\, y_b}{I_{yy}} = -8.000 \text{ m}^{-1}$$

The four inequalities for constructing Magnel diagram are

$$\frac{1}{F_0} \geq \frac{1+\alpha_t e_0}{A\bar{f}_t - \alpha_t M_g} \tag{8}$$

$$\frac{1}{F_0} \leq \frac{1+\alpha_b t e_0}{A\bar{f}_c - \alpha_b M_g} \tag{9}$$

$$\frac{1}{F_e} \leq \frac{1+\alpha_t e_0}{A\bar{f}_c - \alpha_t M_t} \tag{10}$$

$$\frac{1}{F_e} \geq \frac{1+\alpha_b e_0}{A\bar{f}_t - \alpha_b M_t} \tag{11}$$

Inclusion of values of allowable stresses, moment and area in the unequal equations give

$$\frac{1}{F_0} \leq \frac{1+8.000e_0}{-530.4-3557.3} \tag{12}$$

$$\frac{1}{F_0} \geq \frac{1-8.000e_0}{6750.0+3557.3} \tag{13}$$

$$\frac{1}{F_e} \geq \frac{1+8.000e_0}{6750.0-7926.0} \tag{14}$$

$$\frac{1}{F_e} \leq \frac{1-8.000e_0}{-530.4+7926.0} \tag{15}$$

To obtain a minimal prestressed force and maximum eccentricity, equations (14) and (15) provide

$$F_0 = 3840 \text{ kN}, \ F_e = 0.80x \ 3840 = 3070 \text{ kN}; \ e_0 = -0.200 \text{ m}$$

The tension at stage 3 (the dead loads, i.e., self weight of the girder and the weight of superimpose components are active) is

$$f_{ct} = \frac{3840}{0.30} + \frac{(3840)(-0.200)(0.375)}{0.01406} + \frac{(444.66)(0.375)}{0.01406}$$

$$= 12{,}80 - 20.48 + 11.86 = 4.18 \geq -1{,}768 \text{ MPa (ok)}$$

$$f_{cc} = \frac{3840}{0.30} - \frac{(3840)(-0.200)(0.375)}{0.01406} - \frac{(444.66)(0.375)}{0.01406}$$

$$= 12{,}80 + 20.48 - 11.86 = 21.42 \leq 22.50 \text{ MPa (ok)}$$

and the tension at stage 4 (service stage, all of external forces active) is

$$f_{ct} = \frac{3070}{0.30} + \frac{(3070)(-0.200)(0.375)}{0.01406} + \frac{(990.76)(0.375)}{0.01406}$$

$$= 10.23 - 16.38 + 26.42 = 20.27 \ \leq 22.50 \text{ MPa} \ \text{(ok)}$$

$$f_{cc} = \frac{3070}{0.30} - \frac{(3070)(-0.200)(0.375)}{0.01406} - \frac{(990.76)(0.375)}{0.01406}$$

$$= 10.23 + 16.38 - 26.42 = 0.19 \ \geq -1.768 \text{ MPa} \ \text{(ok)}$$

Now, the location of the prestressed cable is checked,

$$e \leq -k_t - \frac{M_t}{F_e} = 0.125 - \frac{990.76}{3070} = \ -0.198 \text{ m}$$

while the lower limit is

$$e \geq \ k_b - \frac{M_g}{F_0} = -0.125 - \frac{444.66}{3840} = -\ 0.240 \text{ m}$$

So that the location of the cable meets the requirements.

(6) The use of tendon

The total initial force used is

$$F_0 = 3840 \text{ kN}$$

The allowable stress in the strand is

$$\bar{f_s} = 0.82\ f_{py} = 0.82 \text{ x } 1680 = 1377.6 \text{ MPa}$$

$$\text{or } \bar{f_s} = \ 0.74\ f_{pu} = 0.74\ x\ 1860 = 1376.4 \text{ MPa (controlled)}$$

so, the strand area needed is

$$A_p = \frac{F_0}{\bar{f_s}} = \frac{3840}{1376.4x1000} = \ 0.0027907 \text{ m}^2 \ = \ \ 2790.7 \text{ mm}^2$$

If the *D*13 strand is used, then $a_p \ = 98.7 \text{ mm}^2$ and the total number of strands needed is

$$n = \frac{A_p}{a_p} = \frac{2790.7}{98.7} = 28.27 \cong 29 \text{ strands}$$

(7) The reinforcement of shear

The upward uniformly distributed force due to the prestressed force is

$$q_F = -\frac{F_e e_0}{L^2} = \frac{8\ x\ 3070x(-0.200)}{(15)^2} \ = 21.83 \text{ kNm}$$

which gives rise to a downward shearing force

$$V_F = -\frac{1}{2}x21.83x15 = -163.73 \text{ kN}$$

so that the total shear force become

$$V_t = V_g + V_s + \ V_\ell + V_p + V_F = 69.86 \text{ kN}$$

The average shear stress becomes

$$\tau_0 = \frac{V_t}{b\ d} = \frac{69.86\ kN}{(0.4\ m\ )(0.75m)} = 0.233 \text{ MPa}$$

The tension of the stirrup permit is $\bar{f_v} = \frac{250}{1.5} = 166.7$ MPa, the area of stirrup is $A_v = 1.327x10^{-4}\text{m}^2$, and spaces s_v, then for stirrup double look obtained

$$\tau_0\, s_v\, b \;=\; 2\,\frac{\sqrt{2}}{2}\,A_v\,\bar{f}_v \tag{16}$$

Thus, from above equations can be obtained

$$s_v = \frac{A_v}{b\;d}\frac{\bar{f}_v}{\tau_0} = \frac{1.327x10^{-4}x166.7}{0.4x0.75x0.233} = 0.316\text{ m}$$

Use stirrup $D13$ – 250. All design results are shown in Table 2.

Table 2 Prestressing force, eccentricity, tendon and stirrup

Item	Quantity
Initial prestressing force	3840 kN
Maximum eccentricity	− 0.200 m
Tendon	29 $D13$
Stirrup	$D13 - 250$

3.2 Design of prestressed concrete beam with shifting of support

To minimize the maximum positive moment in the middle of the span, both positions are shifted as far as a equal to the inside, such as that the absolute value of the negative moment above the support, is equal to the absolute value of the positive moment in the middle of the span (Fig. 2). Since the absolute value of the maximum negative moment is equal to the maximum positive moment, then the design is enough to do for the positive moment only. The height of the beam is estimated at $d = \frac{L}{20} = \frac{15}{20} = 0.75$ m and width $b = \frac{L}{50} = \frac{15}{50} = 0.40$ m. The weights themselves, the superimpose dead load and the live loads are as follows.

- Self-weight of the girder : $q_g = 0.40\;x\;0.75\;x\;25.00 = 7.500$ kN/m
- The superimpose dead loads : $q_s = (0.20x\;25 + 0.075\;x\;22)\;\;x\;1.25 \;\;= 8.313$ kN/m
- Live load (uniform) : $q_\ell = 9\left(0.50 + \frac{15}{30}\right)x1.25 = 11.250$ kN/m
- Concentrated : $P_\ell = 49\;x\;1.25 = 61.250$ kN

so that

$$q_d = \;15.813\text{ kN/m}$$
$$q_t \;= 27.063\text{ kN/m}$$

The negative moment above the support becomes

$$M^- \;= -\tfrac{1}{2}q_t\,a^2 -\; P_\ell\,a \tag{17}$$

and positive moments in the middle of the span become

$$M^+ \;= +\tfrac{1}{8}q_t\,(L-2a)^2 - \tfrac{1}{2}q_t\,a^2 -\; P_\ell\,a + \tfrac{1}{4}P_\ell\,(L-2a) \tag{18}$$

The optimum design is obtained if the absolute value of both moments is the same, so

$$+\tfrac{1}{8}q_t\,(L-2a)^2 - \tfrac{1}{2}q_t\,a^2 -\; P_\ell\,a \;+\; \tfrac{1}{4}P_\ell\,(L-2a) = +\tfrac{1}{2}q_t\,a^2 +\; P_\ell\,a \tag{19}$$

Write down

$$P_\ell\,L = x,\;\; q_tL^2 = 5.3648\,x;\;\; P_\ell\,a = \alpha\,x \tag{20}$$

so that the equation can be written as

$$+0.6706\,(1-2\alpha)^2 - 5.3648\,\alpha^2 -\; 2.500\,\alpha + 0.2500 \;= 0 \tag{21}$$

or

$$\alpha^2 + 1.9320\,\alpha - 0.3432 = 0 \tag{22}$$

with root benefits

$$\alpha = 0.1638$$

that gives

$$\alpha = 01638\,L = 2.457 \approx 2.500 \text{ m}$$

Thus, the moment becomes

$$M^+ = M^- = 274.688 \text{ kNm}$$

Note if both pedestals are still not shifted inward, then the maximum moment becomes

$$M^+ = 0.125\,q\,L^2 = 818.44 \text{ kNm}$$

whose magnitude is almost three times the maximum moment after the shift.

(1) External force and internal force

(a) External force

- Self-weight of the girder : $q_g = 0.40\ x\ 0.75\ x\ 25.00 = 7.500 \text{ kN/m}$
- The dead load of superimpose : $q_s = (0.20x\ 25 + 0.075\ x\ 22)\ x\ 1.25 = 8.313\,\frac{\text{kN}}{\text{m}}$
- Live load (uniform) : $q_\ell = 9\left(0.50 + \frac{15}{30}\right)x1.25 = 11.250 \text{ kN/m}$
- Concentrated : $P = 49\ x\ 1.25 = 61.250 \text{ kN}$

(b) Internal force

- Moment:
  - Girder moment : $M_g = \frac{1}{2}q_g a^2 = 23.438$ kNm
  - Superimpose moments : $M_s = \frac{1}{2}q_s a^2 = 25.978$ kNm
  - Live load (uniform) : $M_\ell = \frac{1}{2}q_\ell a^2 = 35.156$ kNm
  - Live point load moment : $M_p = Pa = 153.125$ kNm
  - Dead load : $M_d = 49.416$ kNm
  - Total load : $M_t = 237.697$ kNm

- Shear force on the pedestal:
  - As a result of girder weight : $V_g = \frac{1}{2}q_g L = 56.250$ kN
  - Result of superimpose : $V_s = \frac{1}{2}q_s L = 62.348$ kN
  - Result of the live load evenly : $V_\ell = \frac{1}{2}q_\ell L = 84.375$ kN
  - Result of the centralized of live load : $V_p = \frac{1}{2}P = 30.625$ kN

Obtained and computed data are given in Table 3.

Table 3 Loads, moments and shear forces, for the case of the shifting of supports

Case	Loading	*M* (kNm)	*V* (kN)
Beam weight	7.500 kN/m	23.438	56.250
Superimpose	8.313 kN/m	25.978	62.330
Uniform live	11.250 kN/m	35.156	84.375
Point live	61.250 kN	153.125	30.625
Total	-	237.697	

(2) Determination of prestressed force and eccentricity

Inclusion of values of allowable stresses, moment and area in the unequal equations give

$$\frac{1}{F_0} \geq \frac{1+8.000e_0}{-530.4-395.3} \tag{23}$$

$$\frac{1}{F_0} \leq \frac{1-8.000e_0}{6750.0+395.3} \tag{24}$$

$$\frac{1}{F_e} \leq \frac{1+8.000e_0}{6750.0-1901.6} \tag{25}$$

$$\frac{1}{F_e} \geq \frac{1-8.000e_0}{-397.8+1901.6} \tag{26}$$

The decisive ones are equations (25) and (26), which give

$$\frac{1+8.000e_0}{6750.0-1901.6} = \frac{1-8.000e_0}{-397.8+1901.6} \tag{27}$$

so that it is obtained

$$F_0 = 1555 \text{ kN}, \; F_e = 0.80x \; 1555 = 1245 \text{ kN}; \; e_0 = -0.125 \text{ m}$$

The stress condition at stage 3 is

$$f_{ct} = \frac{1555}{0.30} + \frac{(1555)\,(-0.125)(0.375)}{0.01406} + \frac{(49.416)(0.375)}{0.01055}$$

$$= 5.18 - 5.18 + 1.32 = 1.32 \; \geq -1.768 \text{ MPa (ok)}$$

$$f_{cc} = \frac{1555}{0.30} = \frac{(1555)\,(-0.125)(0.375)}{0.01406} - \frac{(49.416)(0.375)}{0.01055}$$

$$= 5.18 + 5.18 - 1.32 = 9.04 \; \leq 22.5 \text{ MPa (ok)}$$

and the stress condition at stage 4 is

$$f_{ct} = \frac{1245}{0.30} + \frac{(1245)\,(-0.125)(0.375)}{0.01406} + \frac{(237.697)(0.375)}{0.01055}$$

$$= 4.15 - 4.15 + 6.34 = 6.34 \; \leq 22.5 \text{ MPa (ok)}$$

$$f_{cc} = \frac{1245}{0.30} - \frac{(1245)\,(-0.125)(0.375)}{0.01406} \frac{(237.697)(0.375)}{0.01055}$$

$$= 4.15 + 4.15 - 6.34 = 1.96 \; \geq -1.768 \text{ MPa (ok)}$$

Now check of cable lay out as follows. The upper limit is

$$e \leq k_t - \frac{M_t}{F_e} = 0.125 - \frac{237.697}{1245} = -0.066 \text{ m}$$

and lower limit

$$e \geq k_b - \frac{M_g}{F_0} = -0.125 - \frac{49.416}{1555} = -0.157 \text{ m}$$

Thus, the cross section has a sufficient height for the placement of tendons.

(3) The use of tendon

The total initial force used is

$$F_0 = 1555 \text{ kN}$$

The allowable stress in the strand is

$$\bar{f_s} = 0.82\, f_{py} = 0.82 \text{ x } 1680 = 1377.6 \text{ MPa}$$

$$\text{or } \bar{f_s} = 0.74\, f_{pu} = 0.74\, x\, 1860 = 1376.4 \text{ MPa (controlled)}$$

so, the strand area needed is

$$A_p = \frac{F_0}{\bar{f}_s} = \frac{1555}{1376.4x1000} = 0.00113 \text{ m}^2 = 11229.76 \text{ mm}^2$$

If the $D13$ strand is used, then $a_p = 98.7 \text{ mm}^2$ and the total number of strands needed is

$$n = \frac{A_p}{a_p} = \frac{1129.76}{98.7} = 11.446 \cong 12 \text{ strands}$$

(4) The shearing force

For shear design, the shear force resulting from prestressing forces is determined. The upward even force due to the prestressed force is

$$q_F = -\frac{F_e e_0}{L^2} = -\frac{8 \; x \; 1245x(-0.125)}{(10)^2} = 12.45 \text{ kNm}$$

so that the shear force due to the prestressed force becomes

$$V_F = -\frac{1}{2}x12.45x15 = -93.375 \text{ kN}$$

and the total shear force becomes

$$V_t = V_g + V_s + V_\ell + V_p + V_F = 140.223 \text{ kN}$$

The average shear stress becomes

$$\tau_0 = \frac{V_t}{b\,d} = \frac{161.626 \text{ kN}}{(0.3 \text{ m})(0.75 \text{ m})} = 0.718 \text{ MPa}$$

The tension of the stirrup permit is $\bar{f}_v = \frac{250}{1.5} = 166.7$ MPa, the area of stirrup is $A_v = 1.327x10^{-4}\text{m}^2$, and spaces s_v, then for stirrup double look obtained,

$$s_v = \frac{A_v}{b\,d}\frac{\bar{f}_v}{\tau_0} = \frac{1.327x10^{-4}x166.7}{0.3x0.75x0.718} = 0.137 \text{ m}$$

Use stirrup $D13 - 125$. All design results are shown in Table 4.

Table 4 Prestressing force, eccentricity, tendon and stirrup

Item	Quantity
Initial prestressing force	$1555\ kN$
Maximum eccentricity	$-\ 0.125\ m$
Tendon	$12\ D13$
Stirrup	$D13 - 125$

3.3 Design of prestressed concrete beam with enlarged moment inertia

In this section, construction cost reduction is carried out by the enlargement of the moment of inertia of the section. However, the area and the height of the section are kept constant, so that the reduction of construction cost is due to the enlargement of the moment of inertia only.

(1) The dimension of the section

In this case, the section is made of a hollow section. The height and the area of the section is kept constant, i.e., $d = 0.75$ m, $A_c = 0.30 \text{ m}^2$. The thickness of the flange and the web are taken as $t = 0.15$ m, so

$$0.75 \; x \; b - (0.75 - 2 \; x \; 0.15) \; x \; (b - 2 \; x \; 0.15) = 0.30$$

Hence, $b = 0.55$ meters. The moment of inertia and the Kern limit of area are

$$I_{zz} = \frac{1}{12}x \; 0.55 \; x \; (0.75)^3 - \frac{1}{12}x \; 0.25 \; x \; (0.45)^3 = 0.017438 \text{ m}^4$$

$$W_{zz} = \frac{I_{zz}}{\frac{d}{2}} = \frac{0.017438}{0.375} = 0.046501 \text{ m}^3$$

$$k_t = -k_b = \frac{W_{zz}}{A_c} = \frac{0.046501}{0.300} = 0.155 \text{ m}$$

and

$$\alpha_t = \frac{1}{k_t} = +6.4516 \ \text{m}^{-1}; \ \alpha_b = \frac{1}{k_b} = -6.4516 \ \text{m}^{-1}$$

The moment of inertia is increased from 0.0140625 to 0.017438 m^4 (increased 24 %) and the Kern limit increased from 0.125 to 0.155 m (also increased 24 %).

(2) External force and internal force

(a) External Force

- ❑ Self-weight of the girder :$q_g = 0.30 \ x\ 25.00 = 7.500 \text{ kN/m}$
- ❑ The dead load of superimpose :$q_s = (0.20x\ 25 + 0.075\ x\ 22)\ x\ 1.25 = 8.313 \frac{\text{kN}}{\text{m}}$
- ❑ Live load (uniform) : $q_\ell = 9\left(0.50 + \frac{15}{30}\right)x1.25 = 11.250 \text{ kN/m}$
- ❑ Concentrated : $P = 49\ x\ 1.25 = 61.250 \text{ kN}$

(b) Internal Force

- Moment:
  - ❑ Moment of self-weight girder : $M_g = \frac{1}{8} q_g L^2 = 210.94 \text{ kNm}$
  - ❑ Dead superimpose : $M_s = \frac{1}{8} q_s L^2 = 233.72 \text{ kNm}$
  - ❑ Live uniform load moment : $M_\ell = \frac{1}{8} q_\ell L^2 = 316.41 \text{ kNm}$
  - ❑ Live point load moment : $M_p = \frac{1}{4} P\ L = 229.69 \text{ kNm}$
  - ❑ Total moment : $M_t = 990.76 \text{ kNm}$
  - ❑ Dead load moment : $M_d = 444.66 \text{ kNm}$
- Pedestal shear forces:
  - ❑ Shear of the girder's self-weight : $V_g = \frac{1}{2} q_g L = 56.25 \text{ kN}$
  - ❑ Dead superimpose : $V_s = \frac{1}{2} q_s L = 62.33 \text{ kN}$
  - ❑ Live uniform load moment : $V_\ell = \frac{1}{2} q_\ell L = 84.38 \text{ kN}$
  - ❑ Live point load moment : $V_p = \frac{1}{2} P = 30.63 \text{ kN}$

Loads, moments, shear for all loading cases are given in Table 5.

Table 5 Loads, moments and shear forces, for the case of the enlarged moment inertia

Case	Loading	*M* (kNm)	*V* (kN)
Beam weight	7.500 kN/m	210.94	56.25
Superimpose	8.313 kN/m	233.72	62.33
Uniform live	11.250 kN/m	316.41	84.38
Point live	61.250 kN	229.69	30.63
Total	-	990.76	

(3) Determination of prestressed force and eccentricity

The inequality in prestressed force and eccentricity is give in equations (8-11). Then, Inclusion of values of allowable stresses, moment and area in the unequal equations give

$$\frac{1}{F_0} \geq \frac{1+6.45160e_0}{-530.4-2868.8} \tag{28}$$

$$\frac{1}{F_0} \leq \frac{1-6.4516e_0}{6750.0+2868.8} \tag{29}$$

$$\frac{1}{F_e} \leq \frac{1+6.4516e_0}{6750.0-6392.0} \tag{30}$$

$$\frac{1}{F_e} \geq \frac{1-6.4516e_0}{-530.4+6392.0} \tag{31}$$

The decisive ones are equations (30) and (31), which give

$$\frac{1+6.4516e_0}{6750.0-6392.0} = \frac{1-6.4516e_0}{-530.4+6392.0} \tag{32}$$

so that the prestressing force and the eccentricity become

$$F_0 = 3100 \text{ kN},\ F_e = 0.80x\ 3100 = 2475 \text{ kN};\ e_0 = -0.245 \text{ m}$$

The stress condition at stage 3 is

$$f_{ct} = \frac{3100}{0.300} + \frac{(3100)\ (-0.245)(0.375)}{0.017438} + \frac{(444.66)(0.375)}{0.017438}$$

$$= 10.33 - 16.33 + 9.56 = 3.56\ \geq -1.768 \text{ MPa (ok)}$$

$$f_{cb} = \frac{3100}{0.300} - \frac{(3100)(-0.245)(0.375)}{0.017438} - \frac{(444.66)(0.375)}{0.017438}$$

$$= 10.33 + 16.33 - 9.56 = 17.10\ \leq 22.50 \text{ MPa (ok)}$$

and the stress condition at stage 4 is

$$f_{ct} = \frac{2475}{0.300} + \frac{(2475)\ (-0.245)(0.375)}{0.017438} + \frac{(990.76)(0.375)}{0.017438}$$

$$= 8.25 - 13{,}04 + 21.31 = 16.52\ \cong 22.50 \text{ MPa (ok)}$$

$$f_{cb} = \frac{2475}{0.300} - \frac{(2475)(-0.245)(0.375)}{0.017438} - \frac{(990.76)(0.375)}{0.017438}$$

$$= 8.25 + 13{,}04 - 21.31 = -0.02\ \ \geq -1.768 \text{ MPa (ok)}$$

Now check of cable lay out as follows. The upper limit is

$$e \leq k_t - \frac{M_t}{F_e} = 0.155 - \frac{990.76}{2475} = -0.245 \text{ m}$$

and lower limit

$$e \geq\ k_b - \frac{M_g}{F_0} = -0.155 - \frac{444.66}{3100} = -0.298 \text{ m}$$

Thus, the cross section has a sufficient height for the placement of tendons.

(4) The use of tendon

The total initial force used is

$$F_0 = 3100 \text{ kN}$$

The allowable stress in the strand is

$$\bar{f_s} = 0.82\ f_{py} = 0.82 \text{ x } 1680 = 1377.6 \text{ MPa}$$

$$\text{or } \bar{f_s} =\ 0.74\ f_{pu} = 0.74\ x\ 1860 = 1376.4 \text{ MPa (controlled)}$$

so, the strand area needed is

$$A_p = \frac{F_0}{\bar{f}_s} = \frac{3100}{1376.4x1000} = 0.0022523 \text{ m}^2 = 2252.3 \text{ mm}^2$$

If the $D13$ strand is used, then $a_p = 98.7 \text{ mm}^2$ and the total number of strands needed is

$$n = \frac{A_p}{a_p} = \frac{2252.3}{98.7} = 22.81 \cong 23 \text{ strands}$$

(5) The shearing force

For shear design, the shear force due to prestressing forces is calculated. The upward uniform force due to the prestressed force is

$$q_F = -\frac{F_e e_0}{L^2} = -\frac{8\ x\ 2475x(-0.245)}{(15)^2} = -21.56 \text{ kNm}$$

so that the shear force due to the prestressed force becomes

$$V_F = -\frac{1}{2}x21.56x15 = -161.000 \text{ kN}$$

and the total shear force becomes

$$V_t = V_g + V_s + V_\ell + V_p + V_F = 72.590 \text{ kN}$$

The average shear stress becomes

$$\tau_0 = \frac{V_t}{b\ d} = \frac{72.590\ kN}{(0.25 \text{ m})(0.75\text{m})} = 0.387 \text{ MPa}$$

The tension of the stirrup permit is $\bar{f}_v = \frac{250}{1.5} = 166.7$ MPa, the area of stirrup is $A_v = 1.327x10^{-4}\text{m}^2$, and spaces s_v, then for stirrup double look obtained

$$s_v = \frac{A_v\ \bar{f}_v}{b\ d\ \tau_0} = \frac{1.327x10^{-4}x166.7}{0.3x0.75x0.718} = 0.137 \text{ m}$$

Use stirrup $D13 - 125$. All design results are shown in Table 6.

Table 6 Prestressing force, eccentricity, tendon and stirrup

Item	Quantity
Initial prestressing force	3100 kN
Maximum eccentricity	− 0.245 m
Tendon	23 $D13$
Stirrup	$D13 - 125$

Data obtained for the three cases is tabulated in Table 7. Based on the results out of the optimization process, some aspects in the comparison are given as follows. The shifting of the supports to an amount of 2.500 meters from the total span 15.000 meters, reduces the positive maximum moment from 990.76 kN-m to 237.697 kN-m, it is about 23.99 % of the original moment. Just by shifting the supports, the reduction in the moment magnitude is about 76.01 %. The reduction in the strands used is reduced from 29 strands to 12 strands in shifting support case, the reduction is about 58.62 %, and 23 strands in the case of enlarged moment of inertia, the reduction is about 20.69 %. The stirrup in case of non-optimized design, the stirrup use is $D13$-250, in shifting support case is $D13$-125.

Table 7 Comparison of data obtained in the design

Item	No optimization	Support shift	Enlargement of the moment of inertia
Prestressing	3840 kN	1555 kN	3100 kN
Eccentricity	− 0.200 m	− 0.125 m	− 0.245 m
Tendon	$29D13$	$12D13$	$23D13$
Stirrup	$D13 - 250$	$D13 - 125$	$D13 - 125$

4. Conclusions

This study has proposed two approaches in minimization or the optimization of the design of concrete prestressing system, namely the shifting of the beam supports and the enlargement of the moment of inertia of the section in order to reduce the construction cost of the prestressing concrete beam. Based on the findings in this study, there are several conclusions that can be drawn as follows. This study indicates that the two methods, namely the shifting of the support of the beam and the reshaping the concrete section perform well in the reduction of construction cost of the prestressing concrete beam. The main reduction in the strands used is reduced by about 58.62 %, in the case of the shifting support and by about 20.69 % in the case of the enlarged moment of inertia. For continuing research, some proposals and suggestions are derived as follows. The overall differences in cost should be investigated due to the shifting of the support. For example, if the shifting of the support of beam on the bridge across the river, creates the need of providing extra piers inside the river. Some other cheaper optimization procedure may also be investigated. For example, the use of partial prestressing force.

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