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Premium Calculations with Long Term Care Insurance using Markov Chain Method for Human Immunodeficiency Virus Inpatients

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Abstract— Indonesia has a high prevalence of HIV, which is inversely proportional to its insurance product propolis. Life insurance offers economic protection by providing payment upon the death of the policyholder. Long Term Care (LTC) insurance, a type of health insurance, guarantees care and health insurance for the elderly. This study calculates LTC insurance premiums with annuities as rider benefits using a multi-status model that includes health status, HIV, and death. Using Markov Chain modeling and data from Malang city in 2022, we determine the premium based on transition probabilities. This case study involves a 20-year-old man with 5 years of LTC coverage, a benefit value of IDR100,000,000, and an interest rate of 5%. The premium varies by age and gender: at age 20, male pay Rp 2,719,241 and female Rp 2,701,893, at age 21, male pay Rp. 5,762,095 and female pay Rp.5,746,178. At age 22, male pay at Rp. 6,203,680 and females pay Rp.6,189,454 while at age 23, male pay Rp. 6,640,674 and females pay 6,628,146. And male at 24, pay Rp. 7,066,949 while females pay Rp. 7,012,138 and at age 25, male pay Rp 7,068,616 and females Rp 7,029,781. The results of this calculation show that the more age increases, the greater the net premium and also the male net premium is greater than the female net premium.

Keywords— Human Immunodeficiency Virus (HIV); Markov Chain; Long Term Care Insurance; Net Premium.

I. INTRODUCTION

Life insurance protects against financial losses from events like illness, death, old age, and accidents by having people pay premiums, and a multi-state model using Markov chains helps predict changes in health status to calculate these premiums. HIV has significantly impacted Indonesia, especially among children and productive-aged adults, making premium calculations for HIV patients crucial to manage their risks. Using data from the 2022 Malang city health profile and linear interpolation to calculate prevalence, this study employs a multi-state model to determine term life insurance premiums based on age, policy term, and interest rates.

Research on Long Term Care (LTC) insurance has been conducted by several researchers. Edy Karyady (2022) in "Calculation of life insurance premiums by Markov chain application for heart disease patients in West Kalimantan" shows the dependence of premiums on age and gender [1]. Chrysmandini Pulung Gumauti (2016) in "Calculating long-term care premiums for multi-status models" discusses the calculation of LTC premiums on annuity products as rider benefits with multi-status models and highlights the impact of starting age on the net annual premium amount [2]. Nurmaulia Ningsih (2019) in "A multi-status model for determining health insurance for heart disease patients" discusses the calculation of LTC premiums on annuity products as rider benefits with a multi-status model and highlights the role of interest rates in premium calculations [3]. Adithya Ronnie Effendie (2009) in "Valuation of Long-term Care (LTC) Health Insurance Contract Using Multistate Status" found that Markov process can be used to model multi-status insured phenomenon, although some assumptions are required [4]. Ningsih et al. (2022) in "multi-state model for calculating of long-term care insurance product premium in Indonesia" found that higher interest rates result in cheaper premiums [5].

There are several gaps between this research and other research such as the limited focus, health condition variability, focuses gender and age, and different the use of data variables. In this data used HIV data from the city of Malang in 2022, used interest rate 5% and death benefit values Rp.100.000.000 with 5 years of maximum treatment period.

The study aims to determine the net premium amount for long-term health insurance to ensure insurance companies can cover claims without incurring losses. The multi-state model provides a structured method to calculate premiums, effectively managing the financial risks associated with HIV-related health conditions, and can be adapted for various health conditions and demographics, offering a valuable tool for the insurance industry

II. LITERATURE REVIEW

A. Linear Interpolation

Interpolation is a technique used to estimate the value of a function at specific points, particularly when the function's graph passes through a set of known points. These points can be experimental results or values derived from a known function. Linear interpolation, specifically, calculates the value between two known points using a linear equation. [6]

An illustration of a linear interpolation equation that passes through 2 points, namely and:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \qquad 1$$

B. Stochastic Process and Markov Chain

A stochastic process is a collection of random variables defined in probability space, along with values R. [7]

Let $\{X(t), t \in T\}$ be a family of random variables, where T refers to the process index or parameter space, which is often a subset of R. The set of possible values of a random variable is often referred to as the state space of the process, and is denoted by S.

Stochastic has some types:

1. Markov process: a stochastic process in which the future of the system depends only on its current state and not on the past state.
2. Brownian Process: A continuous stochastic process used to model continuous random movements, such as stock price movements in finance
3. Poisson process: A stochastic process that describes events that occur randomly in time, such as customer arrivals in a queue.

And the other types of stochastic. For this research, the Markov process is the types of stochastic process that we used since the process depends on its current state not on the past state. [8]

A Markov chain is an integer time process $\{Xt, t \geq 0\}$ where each value sampled for each $Xt, t \geq 1$, is contains in a finite set S and depends only on past events only through $Xt - 1$ events. Specifically, for all positive numbers t, and for all $i, j, k, \dots, m$, in the set S.

Moreover, the probability $P\{Xt = j \mid Xt - 1 = i\}$ depends only on i and j and not y and is denoted by

$$P\{Xt = j \mid Xn - 1 = i\} = Pij$$

C. Premium for Long Term Care Insurance Annuity

Long Term Care (LTC) insurance provides income support for chronic disease conditions through annuity benefits and life annuities, covering medical expenses and care in case of disability. LTC insurance is affected by several types of risks, in this case the insurance company only concentrates on risks stemming from morbidity and duration of life (lengths of life).

Benefits LTC insurance benefits can be grouped into three categories:

1. A number of benefits in the form of annuities offered to healthy people

2. A number of benefits in the form of annuities offered to elderly people when they are about to entering or are entering care
3. Repayment of care and medical expenses.

Three states are defined as follows:
1. Healthy occurrence
2. HIV disease
3. Death incident

An illustration of the three-state model can be described as follows: [8]

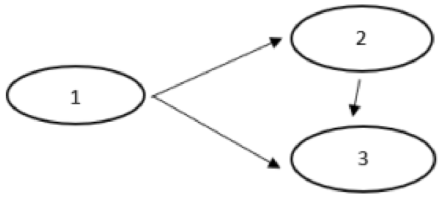

Figure 1. Three- State Model

Denote x as the age of the insured at the policy agreement and $s(x)$ as the survival function in the discrete Markov Chain. The form of the transition matrix for illustration of the three-state model as follows:

$$P_x = \begin{bmatrix} p_x^{11} & p_x^{12} & q_x^{13} \\ 0 & p_x^{22} & q_x^{23} \\ 0 & 0 & 1 \end{bmatrix} \quad 2$$

p_x^{11}: The probability that a person aged x years who is currently in good health (1) will remain in good health (1) at the next time.
p_x^{12}: The value of the probability that a person aged x years who is currently in good health (1) will be in a state of HIV disease (2) the next time.
p_x^{22}: The probability that a person aged x years who is currently in a state of HIV disease (2) will remain in a state of HIV disease (2) in the next time.
q_x^{13}: The probability that a person aged x who is currently health (1) will die (3) in the next time. qx 23

q_x^{23}: The probability that a person aged x who remain in a state of HIV disease (2) will die (3) in the next time
The probability of h steps based on Markov Chain can be written, with:

$$_{\text{h}}p_x^{11} = {}_{\text{h-1}}p_x^{11}\, p_{x+h-1}^{11} \quad 3$$

$$_{\text{h}}p_x^{12} = {}_{\text{h-1}}p_x^{12}\, p_{x+h-1}^{22} + {}_{\text{h-1}}p_x^{11} p_{x+h-1}^{12} \quad 4$$

And, for the matrix form:

$$\cdot_h P_x = \begin{bmatrix} p_x^{11} & p_x^{12} & q_x^{13} \\ 0 & p_x^{22} & q_x^{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_x^{11} & p_{x+1}^{12} & q_{x+1}^{13} \\ 0 & p_{x+1}^{22} & q_{x+1}^{23} \\ 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} p_{x+h-1}^{11} & p_{x+h-1}^{12} & q_{x+h-1}^{13} \\ 0 & p_{x+h-1}^{22} & q_{x+h-1}^{23} \\ 0 & 0 & 1 \end{bmatrix} \quad 5$$

D. Annuity as A Rider Benefit

Annuity as A Rider Benefit, is one of the products of LTC insurance that provides medical care cost benefits during the period of time and death benefits if the insured party dies, either because of his/her illness, or dies without experiencing illness. Annuity as A Rider Benefit products have no transition from stroke disease status to healthy status, as illustrated in Figure 2. The three-state model denotes c as the death benefit, which is the amount of money given when the insured dies. Denoted by b as the annual payment, which is the benefit paid annually when the insured dies paid annually if the insured experiences treatment time. Assume $b = \frac{c}{r}$, where r is the maximum value of the annuity benefit payment time (in years) if the dependent is in treatment. Duration of benefit annuity payment when the insured is in treatment. The duration of benefit annuity payment when the insured is in treatment until death, denoted as f. Annual discount factor, $v = \frac{1}{1+a}$, where a is the interest rate. The net single premium value of LTC insurance is:

$$A_{x:n|}^{LTC} = c \sum_{e=1}^{n} v^{e}{}_{e\text{-}1}P_x^{11} q_{x+e-1}^{13} + b \sum_{e=1}^{n} [v^{e}{}_{e\text{-}1}P_x^{11} P_{x+e-1}^{12} (\ddot{a}_{x+e:r|}^{22} + \sum_{f=1}^{\frac{c}{b}} (c - fb) v^{f}{}_{f\text{-}1}p_{x+e}^{22} q_{x+e+f-1}^{23})] \qquad 6$$

The insurance premium is expected to be paid at the beginning of each year for n years while the insured is still in good health. In [9], the value of the insurance premium is given by

$$P = \frac{A_{x:n|}^{LTC}}{\ddot{a}_{x:n|}^{ij}} \qquad 7$$

III. ANALYSIS AND DISCUSSIONS

A. Determine Linear Interpolation

The HIV prevalence rate data used is basic health research data for Malang city in 2022.

TABLE 1
PERCENTAGE OF HIV PATIENTS IN MALANG CITY 2022

Age group (years-old)	HIV disease (%)
<4	0.8
5-14	0.6
15-19	4.8
20-24	23.3
25-49	61.1
>50	9.4
Male	75.3
Female	24.7
Average	50

Based on data on HIV disease sufferers in Malang City, the value of the interpolation formula in equation 2.1 will be determined. Before determining the value of the interpolation formula, the data will be changed to adjust to the population comparison of Malang city in 2022. In this adjustment, the following data assumptions are used:

Total population (P): 1,000,000 people
Total number of people with HIV disease (J): 50,000 people
Percentage of people with HIV disease in the age group 1-4 years (P_1): 0.8%
Total number of people in the age group 1-4 years (P_2): 100,000 people
HIV disease in the 1-4 years age group (J_1): $50{,}000 \times \frac{80}{100} = 40\ people$
And for the percentage of people with HIV disease aged 1-4 years from the total population (P_3)

$$P_3 = (\frac{40}{1{,}000{,}000})\ X\ 100 = 0.004$$

TABLE 2
PERCENTAGE OF PEOPLE WITH HIV DISEASE

Age group (years-old)	HIV disease (%)
<4	0.0004
5-14	0.0003
15-19	0.0024
20-24	0.01165
25-49	0.03055
>50	0.0047

So, the results of the above calculations will be the data for calculating the prevalence rate later. The prevalence rate calculation of HIV disease sufferers (y) a person at the age of (x) 6 years, then the calculation is

x = 6 years x_1= 5 years x_2= 15 years

With,
y: The result of the prevalence rate of HIV disease patients aged 6 years
y_1: Prevalence rate of HIV disease patients aged 5 years.
y_2: Prevalence rate of HIV disease patients aged 16 years.
So, that the value of the prevalence rate based on the type of HIV disease at the age of 6 years is obtained, namely

1. Prevalence rate of HIV disease patient

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

TABLE 3
PERCENTAGE OF PEOPLE WITH HIV DISEASE

x	x_1	x_2	y_2	y_1
6	5	16	0,0024	0,003
10	9	16	0,0024	0,003
15	14	25	0,01165	0,0024
20	19	25	0,01165	0,0024
21	20	25	0,003055	0,01165
22	21	25	0,003055	0,01165
23	22	25	0,003055	0,01165
24	23	25	0,003055	0,01165

and, for the HIV prevalence rate of the data is

TABLE 4
PREVALENCE RATES OF HIV PATIENT DATA

20	0.01165
21	0.02488
22	0.02677
23	0.02866
24	0.03055
25	0.03055

B. Transition Matrix Compilation

After obtaining the prevalence rate results for each age group, the subsequent step involves computing the one-step transition probability matrix using formula 2.3.
Suppose a man and woman is 20 years old, calculate the one-step transition probability matrix of a man and woman aged 20 years.

1. The probability of death q_x^{13} of a person aged 20 one year later, is $q_x^{13} = 0{,}00049$ for men and $q_x^{13} = 0{,}00027$ for women this value is obtained in the table of mortality age 20 years.
2. The probability of a 20-year-old man in a healthy status (1) becoming a status suffering from HIV disease (2) one year later, namely ($p_{20}^{12} = 0{,}01165$). While, the probability of a 20-year-old woman in a healthy status (1) becoming a status suffering from HIV disease (2) one year later, namely ($p_{20}^{12} = 0{,}01165$).

3. The probability of a 20-year-old man in a healthy state to remain in a healthy state one year later is:

$$p_x^{11} = 1 - p_x^{12} - q_x^{13}$$

$$p_{20}^{11} = 1 - 0{,}01165 - 0{,}00049$$

$$p_{20}^{11} = 0{,}9879$$

While, the probability of a 20-year-old woman in a healthy state to remain in a healthy state one year later is:

$$p_x^{11} = 1 - p_x^{12} - q_x^{13}$$

$$p_{20}^{11} = 1 - 0{,}01165 - 0{,}00027$$

$$p_{20}^{11} = 0{,}9881$$

The probability of a 20-year-old man with a HIV one year later with a comparable contingent value $\eta = 0.05$ is:

$$q_x^{23} = (1+\eta) \times q_x^{13} \qquad ; \eta = 0.05$$

$$q_{20}^{23} = (1+0.05) \times 0{,}00049$$

$$q_{20}^{23} = 0{,}00051$$

While, the probability of a 20-year-old woman with a HIV one year later with a comparable contingent value $\eta = 0.05$ is:

$$q_x^{23} = (1+\eta) \times q_x^{13} \qquad ; \eta = 0.05$$

$$q_{20}^{23} = (1+0.05) \times 0{,}00027$$

$$q_{20}^{23} = 0{,}00028$$

The probability of a 20-year-old man who has a HIV to remain in a HIV illness one year later is

$$p_x^{22} = 1 - q_x^{23}$$

$$p_x^{22} = 1 - 0{,}00051$$

$$p_x^{22} = 0{,}9995$$

While, the probability of a 20-year-old woman who has a HIV to remain in a HIV illness one year later is

$$p_x^{22} = 1 - q_x^{23}$$

$$p_x^{22} = 1 - 0{,}00028$$

$$p_x^{22} = 0{,}9997$$

The following one-step transition probability matrix for a 20-year-old man is obtained:

$$P_{20} = \begin{bmatrix} 0{,}9879 & 0{,}01165 & 0{,}00049 \\ 0 & 0{,}9995 & 0{,}00051 \\ 0 & 0 & 1 \end{bmatrix}$$

While, for a 20-year-old woman is obtained:

$$P_{20} = \begin{bmatrix} 0{,}9881 & 0{,}01165 & 0{,}00027 \\ 0 & 0{,}9997 & 0{,}00028 \\ 0 & 0 & 1 \end{bmatrix}$$

Furthermore, for the calculation of the transition probability matrix h steps for a 20-year-old person are obtained by performing matrix multiplication where $h = 1,2,3,4,5$. Matrix multiplication is taken 5 times transition steps, the 5-step transition probability matrix is as follows:

$${}_1P_{20} = \begin{bmatrix} 0{,}9746 & 0{,}02488 & 0{,}00049 \\ 0 & 0{,}9995 & 0{,}00051 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}_2P_{20} = \begin{bmatrix} 0{,}9727 & 0{,}0268 & 0{,}00049 \\ 0 & 0{,}9995 & 0{,}00051 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}_3P_{20} = \begin{bmatrix} 0{,}9709 & 0{,}0287 & 0{,}00049 \\ 0 & 0{,}9995 & 0{,}00051 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}_4P_{20} = \begin{bmatrix} 0{,}9690 & 0{,}03055 & 0{,}00050 \\ 0 & 0{,}9995 & 0{,}00053 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}_5P_{20} = \begin{bmatrix} 0{,}96893 & 0{,}03055 & 0{,}00052 \\ 0 & 0{,}9995 & 0{,}000546 \\ 0 & 0 & 1 \end{bmatrix}$$

C. Calculation of Long-Term Care Insurance Premium

Calculation of the annual net insurance premium using equation 2.4. Calculation is also made for various values of η (comparison constant), age and premium payment period. With an interest rate value (α), that we used 5%, benefit amount death benefit (c) was IDR 100,000,000, and the premium period of care (r) is 5 years, the calculation of the net premium was based on equation 2.4 as follows:

$$A^{LTC}_{x:n|} = c\sum_{e=1}^{n} v^{e}\,{}_{\text{e-1}}P_x^{11} q_{x+e-1}^{13} + b\sum_{e=1}^{n}[v^{e}\,{}_{\text{e-1}}P_x^{11} P_{x+e-1}^{12}(\ddot{a}_{x+e:r|}^{22} + \sum_{e=1}^{\frac{c}{b}}(c-fb)v^{e}\,{}_{\text{e-1}}p_{x+e}^{22} q_{x+e-1}^{23})]$$

$$A^{LTC}_{x:n|} = 100\,juta\,\sum_{e=1}^{5}(\tfrac{1}{1+0{,}05})^{e}\,{}_{\text{e-1}}P_{20}^{11} q_{20+e-1}^{13} + 20\,juta\sum_{e=1}^{5}[(\tfrac{1}{1+0{,}05})^{e}\,{}_{\text{e-1}}P_{20}^{11} P_{20+e-1}^{12}\ddot{a}_{20+e:5|}^{22}$$

$$A^{LTC}_{x:n|} = 100\,juta\,\sum_{e=1}^{5}\left(\frac{1}{1+0{,}05}\right)^{e}(0{,}9879)(0{,}00049) + 20\,juta\sum_{e=1}^{5}[\left(\frac{1}{1+0{,}05}\right)^{e}(0{,}9879)((0{,}01165)\ddot{a}_{20+e:5|}^{22})$$

$$A^{LTC}_{x:n|} = 100\,juta\,\sum_{e=1}^{5}\left(\frac{1}{1+0{,}05}\right)^{e}(0{,}9879)(0{,}00049) + 20\,juta\sum_{e=1}^{5}[\left(\frac{1}{1+0{,}05}\right)^{e}(0{,}9879)((0{,}233)[\sum_{h=0}^{22}\left(\frac{1}{1{,}05}\right)^{h} p_{20}^{22}])$$

$$A^{LTC}_{x:n|} = 100\,juta\,\sum_{e=1}^{5}\left(\frac{1}{1+0{,}05}\right)^{e}(0{,}9879)(0{,}00049) + 20\,juta\sum_{e=1}^{5}[\left(\frac{1}{1+0{,}05}\right)^{e}(0{,}9879)(0{,}233)[\sum_{h=0}^{22}\left(\frac{1}{1{,}05}\right)^{h}(0{,}9995)]$$

$$A^{LTC}_{x:n|} = Rp14{,}316{,}777$$

And, the net premium is:

$$P\left(A^{LTC}_{20:5|}\right) = \frac{A^{LTC}_{20:5|}}{\sum_{e=1}^{5} v^{h}\, p_{20}^{11}} = \frac{14{,}316{,}777}{5{,}26499} = 2{,}719{,}241$$

Furthermore, the table below lists the results of the premium value for male and female at the age of 20 with an interest rate value (α), that we used 5%, benefit amount death benefit (c) was IDR 100,000,000, and the premium period of care (r) is 5 years.

TABLE 5
RESULT OF THE NET PREMIUM

Age	**Gender**	
	Male	**Female**
20	Rp. 2,719,241	Rp. 2,701,893
21	Rp. 5,762,095	Rp. 5,746,178
22	Rp. 6,203,680	Rp. 6,189,454
23	Rp. 6,640,674	Rp. 6,628,146
24	Rp. 7,066,949	Rp. 7,012,138
25	Rp. 7,068,616	Rp. 7,029,781

The results in the table above show that, the older a person is, the greater the amount of premium paid. The results obtained also show that the value of male premiums will be greater than women.

IV. CONCLUSION

The conclusion Long-term care (LTC) insurance provides long-term health protection and allows for the inclusion of risk factors such as HIV status in premium calculations, as individuals with certain health conditions, like HIV, have different long-term care needs and higher costs. Using a multi-state model, LTC insurance can adjust premiums based on various factors, including health status, HIV status, and mortality. The data components used for each age, including mortality tables and linear interpolation results, affect net premium calculations. As individuals age, premiums increase due to higher chances of death or developing HIV. The n-step transition probability is used to calculate premiums and reserves for a three-condition endowment life insurance product. Results indicate that male premiums are higher than female premiums, influenced by the higher likelihood of men contracting

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