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Joint Life Endowment Insurance Premium Calculation Based on Makeham Mortality Law

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Abstract— This research determined life insurance premiums joint-life endowment insurance, that is the insurance that the benefit is paid if one of the insureds dies or all the insureds survive until the end of the contract. This premium calculation uses one of the famous mortality laws, that is Makeham. Makeham mortality law is an updated law of Gompertz's mortality law where the reason that makes Makeham different from Gompertz is the death calculation factor, because Gompertz only takes the cause of death from the age factor. Even though the death factor can also come from accidents, therefore, Makeham combined the two causes of death so that it is corresponded to the actual factors. The Makeham law will function to create the Makeham assumption mortality table with trials and errors method to construct Indonesia Mortality Table 2019 (TMI IV). This construction requires parameters that must be founded to determine the survival probability (p_x) of the Makeham. After finding the p_x, calculate the annual premium by dividing the net single premium by the life annuity. The joint-life endowment insurance premium calculation in this study uses 10-year protection with a 5.75% for interest rate and Rp1,000,000,000 for the benefit, and based on the research, the annual premium for this married couple is Rp78,016,785, where the premium calculated based on Makeham will get a higher premium than calculated based on TMI 2019, which gets an annual premium Rp77,410,954.

Keywords— Endowment Insurance; Joint-Life Status; Annual Premium; Makeham's Mortality Law

I. INTRODUCTION

Throughout people live in this world, people cannot figure out what will happen later in the future. Every activity people do daily there is a chance to get a risk. In common, risk is characterized as the combination of the probability of an occasion and its negative result [1] therefore there must be an effort to keep people safe and an effort to minimize if people are perceived to have a large impact on losses. Risk is also a hazard that is associated as related to the extent to which people cannot adapt (lack of capacity) to a specific circumstance [2]. Because that, buying products (insurance) from an insurance company is needed because the insurance company will help people to minimize the risk of loss that will impact the main source of life which is financial because insurance companies provide savings and investment benefits for the future.

There are eight different kinds of insurance that can be bought, according to [3]: life, health, vehicle, renter, home, umbrella, disability, and long-term care insurance. Specially for life insurance, there are many benefits provided by this type of insurance such as financial protection for family or heirs, life cover against uncertainly, and can be as an investment. There are several types of life insurance described in [4] include whole life insurance, which provides for payment of one unit at the end of the year of death. Term insurance of duration n that is insurance that pays out only if death occurs within n years. There is also a pure endowment which has an n-year period and only pays out the full insured amount if the policyholder is still living after n years. The last is endowment insurance, which assumes that the payment of the insured amount is payable at the conclusion of the death year.

In addition, there are also types of life insurance based on the number of insureds, which are individual life insurance that involves only one life and combined insurance that can involve several lives. The combined insurance has two categories there are joint life and last survivor. Joint life insurance could be a type of insurance with premiums paid until the first death of one of the two insured people with benefits paid right away and for the last survivor insurance is insurance that will get benefits when the second insured dies. The joint life is more useful because it will help protect two insureds at once by providing that if one of the insureds dies, then the benefits will be given and the policy contract also ends.

So, if an insured has determined the type of insurance, then the insured must pay a premium amount according to the type of insurance or policy chosen. The highly recommended to buy insurance between type of insurance is endowment insurance because the benefits provided are not only based on the event of death, but if the insured survives in the specified year, the insurance company will still give benefits. Despite the endowment insurance is highly recommended, the insured has to pay attention to the premium that will be paid. Many types of companies sell the same type of insurance with the same coverage but have different prices. If the company sells at the cheapest price of the insurance, the insured will think about the ability of the company to cover the risks, and if the company sells at the more expensive price of the insurance, the insured will be interested in buying in another company with the cheapest price and with the same coverage, because that the calculation of premium for the company is necessary to choose the best price which does not make the company incur losses and also make the company able to cover all the risks.

Premiums paid by the insured usually require the calculation of future life expectancy. This calculation can be called a mortality table. Initially, Indonesia already had a mortality table as a reference that launched by AAJI [5] namely Indonesia Mortality Table 2019 (TMI IV) to determine life expectancy in Indonesia. In addition to the mortality table, there is also a mortality law, which is a form of approach to accelerating mortality based on the mortality table. This approach is used because the mortality law has a formula that can explain phenomena that occur practically and efficiently, and it tends to be simpler to estimate parameter's function from mortality data. There are several mortality laws described in [6] there are the De Moivre mortality law (1724), Gompertz (1825), Makeham (1860), and Weibull (1939). Among the several mortality laws, Makeham mortality law is more flexible because Makeham helps to account for death by age and other factors, so that is not only affected by age.

The modified De Moivre method that studied by [7] is a development from the De Moivre method where there is an alpha value as a parameter also by using n-year term insurance with scope and limitation of the research, the cheaper annual premium is Modified De Moivre method with harmonic mean dan the most expensive annual net premium is Modified De Moivre with median alpha. The Gomperz's law that [8] used obtained that the calculation for a single life insurance premium for a lifetime is influenced by parameters on the Gomperz distribution, the age of the insurer, and the interest rate used, as well as the result that there are differences in premium values between men and women, and that is due to differences in the probability of death from the mortality table. Research about Makeham did by [9] obtained if using the acceleration of Makeham Law, the result will increase premium until the last insured death in the case of the last survivor. The last law mortality that studied by [10] was Weibull by using stochastic interest rates CIR and got the result both male and female insureds have the lowest premiums compared to calculations using TMI 2011 and the calculation of endowment life insurance premiums is relatively not much different for female customers in the toddler, child and senior age groups (0-65 years), however, large differences begin to occur at the age of 90 years and above. Based on previous research the researcher is interested in calculating premiums with the type of number insured is joint life because joint life is more useful to buy by cover the two insured with the type of insurance is endowment and also using the Makeham mortality law to determine parameter estimates from TMI IV 2019 mortality rate reference.

The results of this study will also produce different premium prices from previous studies because of the different types of mortality laws, calculation methods, and types of insurance used in this study, especially with previous studies that used Gomperz's law, where Makeham's law is a renewal of the law of mortality from Gompertz's law. In Gomperz's law, death occurs only based on age; therefore, the calculation in Gomperz and Makeham differs by only one variable, which in Makeham's law states that death occurs not only due to age but also by other causes such as accidents. The premium result from Gomperz and Makeham is similar. Still, of course, the premium will be more expensive if the calculation uses Makeham's law, where one more variable is added to the calculation.

This study also provides substantiation of its contribution to theory and practice. This study provides evidence supporting that Makeham's law is a law whose calculation is flexible and easy to apply to the calculation of any type of insurance, and Makeham's law also makes the probability of someone dying higher than the probability of someone being alive because of the additional variable in its calculation (another factor causing death). In addition, it also proved that using the trials and errors method to find Makeham's parameters is an accurate method in which it provides results that are more compatible with life expectancy in Indonesia. Therefore, the contents of this study provide a flexible and easy calculation overview that will help readers understand how to calculate joint-life endowment insurance premiums using Makeham's law based on the state of life expectancy in Indonesia.

II. METHOD

A. Type of The Research

Quantitative research is a type of this research. This research used secondary data from Indonesian Mortality Table 2019 (TMI IV) which was constructed using the Makeham mortality law and used trials and errors for finding the parameters. The results of this study will result in the amount of premiums joint-life endowment life insurance that the insureds must pay using the example of a married couple with a husband's age of 25 years and a wife's age of 25 years old. This research will use Microsoft Excel for data processing and calculation.

B. Data Collection Method

The collecting data is used documentation methods. The documentation method is a method to find data regarding things in the form of books, notes, inscriptions, newspapers, and proceedings. This research used the documentation method by taking secondary data from TMI IV 2019 which can be accessed through the Insurance Asosiasi Asuransi Jiwa Indonesia (AAJI). The interest rate in this research used a 5.75% interest rate (i %) which was taken from Bank Indonesia [11] as of 21st September 2023.

C. Flowchart

This research was carried out using several systematic work steps to obtain optimal results. Research work steps are a series of research procedures and steps in conducting research that are structured in a systematic and directed manner so that the objectives of the research can be achieved. Below are the steps during research in finding the premium from joint-life n-year endowment insurance that can be shown using a flowchart in Figure. 1.

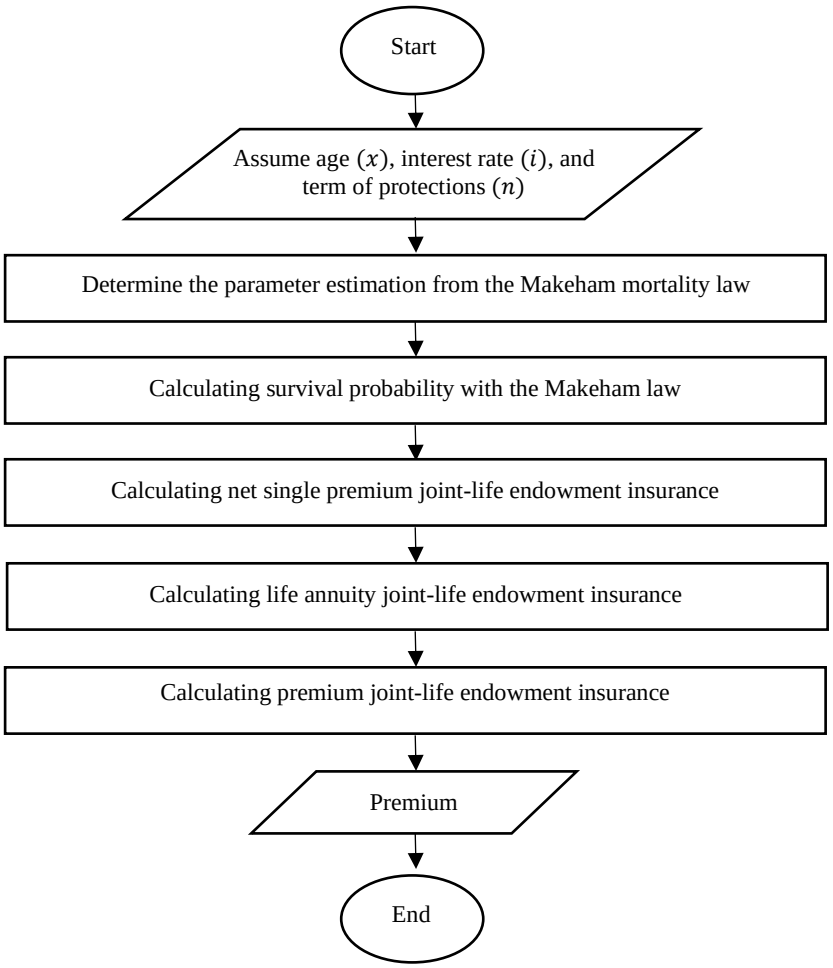

Figure. 1 Flowchart to find premium joint-life endowment insurance

III. RESULTS AND DISCUSSION

This research assumed that married couple, with age for husband and wife are 25 years old, bought a joint-life endowment insurance with 10 years protection and will get Rp1,000,000,000 for benefit. In calculating premium joint-life endowment, mortality table and interest rate is needed. So, this research used secondary data from Indonesian mortality table 2019 (TMI IV) and 5.75% for interest rate (the interest rate is taken from Bank Indonesia as of 21st September 2023) to help in calculations in finding the premium value. Here are the following steps to calculate joint-life endowment insurance based on Makeham mortality law with trials and errors method:

1. Determine the parameter estimation and survival probability from the Makeham mortality law using trials and errors method

Based on the Makeham's interval $0.001 < A < 0.003;\ 10^{-6} < B < 10^{-3}; 1.08 < c < 1.12$, parameters of Makeham can be found by using trials and errors method by conducting out experiments one by one over a range of interval. In this research, the interval that will be used for parameter A will be paired with each interval for parameter c so that it can produce 105 trials and errors experiments per age.

TABLE 1
105 PAIRS OF TRIALS AND ERRORS FOR PARAMETER A AND c

A	c	A	c	A	c	A	c	A	c
0.001		0.001		0.001		0.001		0.001	
0.0011		0.0011		0.0011		0.0011		0.0011	
0.0012		0.0012		0.0012		0.0012		0.0012	
0.0013		0.0013		0.0013		0.0013		0.0013	
0.0014	1.08	0.0014	1.09	0.0014	1.10	0.0014	1.11	0.0014	1.12
0.0015		0.0015		0.0015		0.0015		0.0015	
0.0016		0.0016		0.0016		0.0016		0.0016	
0.0017		0.0017		0.0017		0.0017		0.0017	
0.0018		0.0018		0.0018		0.0018		0.0018	
0.0019		0.0019		0.0019		0.0019		0.0019	
0.002		0.002		0.002		0.002		0.002	
0.0021		0.0021		0.0021		0.0021		0.0021	
0.0022		0.0022		0.0022		0.0022		0.0022	
0.0023		0.0023		0.0023		0.0023		0.0023	
0.0024	1.08	0.0024	1.09	0.0024	1.10	0.0024	1.11	0.0024	1.12
0.0025		0.0025		0.0025		0.0025		0.0025	
0.0026		0.0026		0.0026		0.0026		0.0026	
0.0027		0.0027		0.0027		0.0027		0.0027	
0.0028		0.0028		0.0028		0.0028		0.0028	
0.0029		0.0029		0.0029		0.0029		0.0029	
0.003		0.003		0.003		0.003		0.003	

a) Parameter estimation and survival probability from Makeham mortality law for men
Using the Makeham survival probability formula below

$$p_x = \exp\left[-A - \frac{Bc^x}{\ln c}(c-1)\right] \quad (1)$$

With formula to find parameter B is

$$-\frac{\ln c\left[\frac{ln\, p_x + A}{(c-1)}\right]}{c^x} = B \quad (2)$$

So, the Makeham mortality table for men can find by calculate the survival probability with trials and errors method with the makeham's interval for each parameter $0.001 < A < 0.003;\ 10^{-6} < B < 10^{-3}; 1.08 < c < 1.12$ and choose the minimum result of error between the Makeham mortality and TMI IV. For example, find the Makeham survival probability for Men if age (x) is 35 with parameter A is 0.001, c is 1.08 and p_{35} is 0.99893 (based on TMI 2019).

So, parameter B will be

$$B = -\frac{\ln c\left[\frac{ln\, p_x + A}{(c-1)}\right]}{c^x}$$

$$B = -\frac{0.076961041\left[\frac{(-0.001070573) + 0.001}{(1.08-1)}\right]}{14.78534429}$$

$B = 0.00000459$

Because the B result above is still included in interval B, the B result can be applied to Makeham's survival probability formula then,

$$p_x Makeham = \exp\left[-A - \frac{Bc^x}{\ln c}(c-1)\right]$$

$$p_x Makeham = \exp\left[-0.001 - \frac{(0.00000459)(14.78534429)}{0.076961041}(1.08-1)\right]$$

$$p_x Makeham = 0.99893001297$$

The next step is to compare the value between $p_x Makeham$ and p_x

$Error = p_x Makeham - p_x$

$Error = 0.99893001297 - 0.99893$

$Error = 0.00000001297 \; or \; 0.0000000130$

The error above is an example of a small error, but because the number of trials and errors per age is 105 trials, there are still 104 trials that can be calculated to get a smaller error. The summary of 105 trials that were selected based on the smallest error value for each type of parameter c (1.08; 1.09; 1.10; 1.11; 1.12) can be seen in Table 2.

TABLE 2
COMPARISON OF ERROR RESULTS

$p_x Makeham$	A	B	c	Error
0.99893001297	0.001	0.00000459	1.08	1.30E-08
0.99893000563	0.001	0.00000331	1.09	5.63E-09
0.99893001466	0.001	0.00000239	1.10	1.47E-08
0.99893002847	0.001	0.00000174	1.11	2.85E-08
0.99893001723	0.001	0.00000126	1.12	1.72E-08

From the five trials and errors above, the one that gets th e smallest error is when p_x Makeham is 0.99893000563 where the parameters are $A; 0.001\; B; 0.00000331 \; and \; c; 1.09$. So, p_x Makeham for man when aged 35 is 0.99893000563. Continue the calculations as before until the age of 110 for men until get the survival probability (p_x) Makeham for men as seen in Table 3.

TABLE 3
MAKEHAM SURVIVAL PROBABILITY TABLE FOR MEN

x	$p_x Makeham$	x	$p_x Makeham$	x	$p_x Makeham$
0	0.99594935911	38	0.99861000147	76	0.97631000577
1	0.99899937831	39	0.99845000009	77	0.97262000629
2	0.99899928859	40	0.99827000111	78	0.96870000270
3	0.99899919169	41	0.99807000037	79	0.96307001179
4	0.99899908704	42	0.99784000058	80	0.95482000484
5	0.99899897401	43	0.99759000071	81	0.94473000526
6	0.99899885195	44	0.99730000066	82	0.93268000162
7	0.99899872012	45	0.99698000015	83	0.91772001500
8	0.99899857774	46	0.99662000007	84	0.90522000083
9	0.99899842397	47	0.99623000077	85	0.89535000713
10	0.99899825790	48	0.99582000059	86	0.88467000595
11	0.99899807855	49	0.99539000023	87	0.87302000355
12	0.99899788485	50	0.99492000021	88	0.86053001932
13	0.99899767565	51	0.99444000123	89	0.84729000446
14	0.99899744972	52	0.99391000025	90	0.83341000411
15	0.99899720571	53	0.99333000028	91	0.82009000209
16	0.99899694218	54	0.99273000018	92	0.80610003409
17	0.99899665756	55	0.99211000103	93	0.79126000656
18	0.99899635018	56	0.99153000150	94	0.77549001616
19	0.99899601821	57	0.99102000120	95	0.75874003521
20	0.99899565968	58	0.99061000594	96	0.74285000791
21	0.99899527247	59	0.99029000053	97	0.72581006108
22	0.99899485428	60	0.99001000021	98	0.70751004336
23	0.99899440264	61	0.98976000004	99	0.68785001529
24	0.99899391487	62	0.98954000008	100	0.66669005466
25	0.99899338807	63	0.98929000226	101	0.64837000657
26	0.99899281913	64	0.98896000190	102	0.62868002113
27	0.99899220468	65	0.98854000621	103	0.60750002034

x	$p_x Makeham$	x	$p_x Makeham$	x	$p_x Makeham$
28	0.99899154107	66	0.98801000070	104	0.58473000491
29	0.99899082437	67	0.98740000062	105	0.56027003756
30	0.99899005034	68	0.98671000184	106	0.53398002732
31	0.99898921439	69	0.98595000375	107	0.50571000381
32	0.99898831156	70	0.98515000925	108	0.47533000997
33	0.99898733650	71	0.98426000016	109	0.44267005012
34	0.99898628344	72	0.98330000680	110	0.40756002942
35	0.99893000563	73	0.98223001949	111	0
36	0.99884000295	74	0.98105000099		
37	0.99873000245	75	0.97974000655		

b) Parameter estimation and survial probability from Makeham mortality law for women

Same as men calculation, the Makeham mortality table for women can find by calculate the survival probability with trials and errors method with the makeham's interval for each parameter $0.001 < A < 0.003;\ 10^{-6} < B < 10^{-3}; 1.08 < c < 1.12$ and choose the minimum result of error between the Makeham mortality and TMI IV. The survival probability (p_x) Makeham for women can be seen in Table 4.

TABLE 4
MAKEHAM SURVIVAL PROBABILITY TABLE FOR WOMEN

x	$p_x Makeham$	x	$p_x Makeham$	x	$p_x Makeham$
0	0.99734000002	38	0.99898115864	76	0.98121001051
1	0.99899937831	39	0.99892000130	77	0.97970000691
2	0.99899928859	40	0.99882000248	78	0.97674000405
3	0.99899919169	41	0.99872000026	79	0.97120001669
4	0.99899908704	42	0.99859000072	80	0.96431001662
5	0.99899897401	43	0.99846000323	81	0.95792000704
6	0.99899885195	44	0.99831000473	82	0.95093000775
7	0.99899872012	45	0.99813000023	83	0.94480000240
8	0.99899857774	46	0.99791000164	84	0.93914000637
9	0.99899842397	47	0.99770000122	85	0.93285001205
10	0.99899825790	48	0.99747000173	86	0.92682000184
11	0.99899807855	49	0.99723000003	87	0.91845002039
12	0.99899788485	50	0.99695000029	88	0.90955000752
13	0.99899767565	51	0.99665000146	89	0.89999001452
14	0.99899744972	52	0.99632000051	90	0.89087001049
15	0.99899720571	53	0.99597000037	91	0.88479002120
16	0.99899694218	54	0.99558000111	92	0.87501000042
17	0.99899665756	55	0.99517000046	93	0.86174002323
18	0.99899635018	56	0.99476000095	94	0.84549001500
19	0.99899601821	57	0.99437000171	95	0.82571000763
20	0.99899565968	58	0.99399000286	96	0.80845000793
21	0.99899527247	59	0.99364000008	97	0.79404000221
22	0.99899485428	60	0.99329000002	98	0.77773000903
23	0.99899440264	61	0.99293000091	99	0.76264003922
24	0.99899391487	62	0.99254000110	100	0.74190000974
25	0.99899338807	63	0.99212000077	101	0.71932007552
26	0.99899281913	64	0.99167000062	102	0.69438000061
27	0.99899220468	65	0.99117000341	103	0.66685000459
28	0.99899154107	66	0.99060000138	104	0.63631001305
29	0.99899082437	67	0.98995000081	105	0.60682001571
30	0.99899005034	68	0.98924000373	106	0.57117001570
31	0.99898921439	69	0.98850000766	107	0.53396000821
32	0.99898831156	70	0.98771000966	108	0.49573000294
33	0.99898733650	71	0.98686000556	109	0.45523005573
34	0.99898628344	72	0.98594002420	110	0.41298005657
35	0.99898514614	73	0.98492000264	111	0
36	0.99898391785	74	0.98380001485		
37	0.99898259131	75	0.98257004113		

2. Calculate net single premium joint-life n-year term endowment life insurance $A_{xy:\overline{n}|}$ based on the Makeham assumption mortality table

To calculate annual joint-life endowment insurance premium. the first is calculate the net single premium that is using formula

$$\neg \qquad (3)$$

$$A_{xy:n} = \sum_{k=1}^{n} v^k \, {}_{k-1|}q_{xy} + v^n \, {}_{n}p_{xy}$$

With age of husband 25 years old (x) and wife 25 years old (y) and by using 10-year protection then the formula become

$$A_{25:25:\overline{10}|} = \sum_{k=1}^{10} v^k \, {}_{k-1|}q_{25:25} + v^n \, {}_{n}p_{xy}$$

$$A_{25:25:\overline{10}|} = \sum_{k=1}^{10} v^k \, {}_{k-1|}q_{25:25} + v^{10} \, {}_{10}p_{25:25}$$

Following the formula v with interest rate 5.75% so,

$$v = \frac{1}{1+i} = \frac{1}{1+5.75\%} = \frac{1}{1.0575}$$

To make the calculation easier. The calculation for $A_{25:25:\overline{10}|}$ are separated by two steps:

a. Calculate

$$\sum_{k=1}^{10} v^k \, {}_{k-1|}q_{25:25}$$

$$\sum_{k=1}^{10} \left(\frac{1}{1.0575}\right)^k \left({}_{k-1}p_{25:25} - {}_{k+1-1}p_{25:25}\right)$$

$$\sum_{k=1}^{10} \left(\frac{1}{1.0575}\right)^k \left({}_{k-1}p_{25:25} - {}_{k}p_{25:25}\right)$$

$$\sum_{k=1}^{10} \left(\frac{1}{1.0575}\right)^k \left({}_{k-1}p_{25} \; {}_{k-1}p_{25} - {}_{k}p_{25} \; {}_{k}p_{25}\right)$$

$$\left(\frac{1}{1.0575}\right)^1 \left({}_{1-1}p_{25} \; {}_{1-1}p_{25} - {}_{1}p_{25} \; {}_{1}p_{25}\right) + \cdots$$

$$\ldots + \left(\frac{1}{1.0575}\right)^{10} \left({}_{10-1}p_{25} \; {}_{10-1}p_{25} - {}_{10}p_{25} \; {}_{10}p_{25}\right)$$

Based on Table 3 and Table 4 that are the Makeham mortality table assumption, the survival probability for husband and wife and discount factor (v) in this calculation is following the Table 5.

TABLE 5
TABLE FOR SURVIVAL PROBABILITY AND DISCOUNT FACTOR

$\boldsymbol{v = \left(\frac{1}{1+i}\right)^k}$	$\boldsymbol{x = Husband = 25\ years\ old}$	$\boldsymbol{x = Wife = 25\ years\ old}$
$\left(\frac{1}{1.0575}\right)^1 = 0.945626$	${}_0p_{25} = 1$	${}_0p_{25} = 1$
$\left(\frac{1}{1.0575}\right)^2 = 0.894209$	${}_1p_{25} = 0.998993915$	${}_1p_{25} = 0.998993915$
$\left(\frac{1}{1.0575}\right)^3 = 0.845588$	${}_2p_{25} = p_{25} \cdot p_{26} = 0.997988316$	${}_2p_{25} = p_{25} \cdot p_{26} = 0.997988316$
$\left(\frac{1}{1.0575}\right)^4 = 0.799611$	${}_3p_{25} = {}_2p_{25} \cdot p_{27} = 0.996983161$	${}_3p_{25} = {}_2p_{25} \cdot p_{27} = 0.996983161$
$\left(\frac{1}{1.0575}\right)^5 = 0.756133$	${}_4p_{25} = {}_3p_{25} \cdot p_{28} = 0.995978406$	${}_4p_{25} = {}_3p_{25} \cdot p_{28} = 0.995978406$
$\left(\frac{1}{1.0575}\right)^6 = 0.715019$	${}_5p_{25} = {}_4p_{25} \cdot p_{29} = 0.994974003$	${}_5p_{25} = {}_4p_{25} \cdot p_{29} = 0.994974003$
$\left(\frac{1}{1.0575}\right)^7 = 0.676141$	${}_6p_{25} = {}_5p_{25} \cdot p_{30} = 0.993969899$	${}_6p_{25} = {}_5p_{25} \cdot p_{30} = 0.993969899$

$v = \left(\frac{1}{1+i}\right)^k$	$x = Husband = 25\ years\ old$	$x = Wife = 25\ years\ old$
$\left(\frac{1}{1.0575}\right)^8 = 0.639377$	${}_7p_{25} = {}_6p_{25} \cdot p_{31} = 0.992966040$	${}_7p_{25} = {}_6p_{25} \cdot p_{31} = 0.992966040$
$\left(\frac{1}{1.0575}\right)^9 = 0.604612$	${}_8p_{25} = {}_7p_{25} \cdot p_{32} = 0.991962364$	${}_8p_{25} = {}_7p_{25} \cdot p_{32} = 0.991962364$
$\left(\frac{1}{1.0575}\right)^{10} = 0.571737$	${}_9p_{25} = {}_8p_{25} \cdot p_{33} = 0.990958807$	${}_9p_{25} = {}_8p_{25} \cdot p_{33} = 0.990958807$
	${}_{10}p_{25} = {}_9p_{25} \cdot p_{34} = 0.989955299$	${}_{10}p_{25} = {}_9p_{25} \cdot p_{34} = 0.989955299$

$$\sum_{k=1}^{10} v^k\ {}_{k-1|}q_{25:25} = \left(\frac{1}{1.0575}\right)^1 ({}_0p_{25}\ {}_0p_{25} - {}_1p_{25}\ {}_1p_{25}) + \cdots$$
$$\cdots + \left(\frac{1}{1.0575}\right)^{10} ({}_9p_{25}\ {}_9p_{25} - {}_{10}p_{25}\ {}_{10}p_{25})$$
$$= 0.014896449$$

b. Calculate
$v^{10}\ {}_{10}p_{25:25}$
$v^{10}\ ({}_{10}p_{25} \cdot {}_{10}p_{25})$
Based on Table 5 then,
$v^{10}\ {}_{10}p_{25:25} = (0.571737) \cdot (0.989955299) \cdot (989955299)$
$= 0.560308756$
So, the net single premium of joint-life endowment insurance can be shown,

$$A_{25:25:\overline{10}|} = \sum_{k=1}^{10} v^k\ {}_{k-1|}q_{25:25} + v^{10}\ {}_{10}p_{25:25}$$
$$A_{25:25:\overline{10}|} = 0.014896449 + 0.560308756 = 0.575205205$$

3. Calculate life annuity premium joint-life *n*-year term endowment life insurance $\ddot{a}_{xy:\overline{n}|}$ based on the Makeham assumption mortality law

The second step to find premium is find the life annuity. Life annuity that using for the calculation is life annuity joint-life *n*-year temporary life insurance.

$$\ddot{a}_{xy:\overline{n}|} = \sum_{k=1}^{10} v^k\ {}_kp_{xy}$$
$$\ddot{a}_{25:25:\overline{10}|} = \sum_{k=1}^{10} v^k\ {}_kp_x \cdot {}_kp_y \cdot$$
$$\ddot{a}_{25:25:\overline{10}|} = v^1\ {}_1p_{25} \cdot {}_1p_{25} + \cdots + v^{10}\ {}_{10}p_{25} \cdot {}_{10}p_{25}$$
$$\ddot{a}_{25:25:\overline{10}|} = \left(\frac{1}{1.0575}\right)^1\ {}_1p_{25} \cdot {}_1p_{25} + \cdots + \left(\frac{1}{1.0575}\right)^{10}\ {}_{10}p_{25} \cdot {}_{10}p_{25}$$

Based on Table 5 the life annuity joint-life insurance got the result
$\ddot{a}_{25:25:\overline{10}|} = 7.372839121$

4. Calculate net premium joint-life *n*-year term endowment life insurance $P_{xy:\overline{n}|}$ .based on the Makeham assumption mortality law

The calculation of annual joint-life endowment insurance premium can be shown as,

$$P_{xy:\overline{n}|} = \text{Rp}1{,}000{,}000{,}000\ \frac{A_{xy:\overline{n}|}}{\ddot{a}_{xy:\overline{n}|}}$$

$$P_{25:25:\overline{10}|} = \text{Rp}1{,}000{,}000{,}000 \frac{0.575205205}{7.372839121} = \text{Rp}78{,}016{,}785$$

So, Rp78,016,785 is the annual premium for joint-life endowment insurance based on Makeham assumption mortality table for married couples with Rp1,000,000,000 benefit and 10-year protection.

5. Calculate net premium joint-life n-year term endowment life insurance $P_{xy:\overline{n}|}$ .based on the Indonesian Mortality Table 2019

Same calculation as net premium based on Makeham, so the calculation of annual joint-life endowment insurance based on TMI 2019 can be shown in formula (3) or as,

$$A_{xy:\overline{n}|} = \sum_{k=1}^{n} v^k \, {}_{k-1|}q_{xy} + v^n \, {}_{n}p_{xy}$$

With age of husband 25 years old (x) and wife 25 years old (y) and by using 10-year protection then the formula become

$$A_{25:25:\overline{10}|} = \sum_{k=1}^{10} v^k \, {}_{k-1|}q_{25:25} + v^{10} \, {}_{10}p_{25:25}$$

Following the formula v with interest rate 5.75% so,

$$v = \frac{1}{1+i} = \frac{1}{1+5.75\%} = \frac{1}{1.0575}$$

And for TMI 2019 can be seen below

TABLE 6
INDONESIAN MORTALITY TABLE 2019

Age	Man	Woman	Age	Man	Woman	Age	Man	Woman
0	0.00524	0.00266	38	0.00139	0.00100	76	0.02369	0.01879
1	0.00053	0.00041	39	0.00155	0.00108	77	0.02738	0.02030
2	0.00042	0.00031	40	0.00173	0.00118	78	0.03130	0.02326
3	0.00034	0.00024	41	0.00193	0.00128	79	0.03693	0.02880
4	0.00029	0.00021	42	0.00216	0.00141	80	0.04518	0.03569
5	0.00026	0.00020	43	0.00241	0.00154	81	0.05527	0.04208
6	0.00023	0.00022	44	0.00270	0.00169	82	0.06732	0.04907
7	0.00021	0.00023	45	0.00302	0.00187	83	0.08228	0.05520
8	0.00020	0.00022	46	0.00338	0.00209	84	0.09478	0.06086
9	0.00020	0.00021	47	0.00377	0.00230	85	0.10465	0.06715
10	0.00019	0.00019	48	0.00418	0.00253	86	0.11533	0.07318
11	0.00019	0.00018	49	0.00461	0.00277	87	0.12698	0.08155
12	0.00019	0.00020	50	0.00508	0.00305	88	0.13947	0.09045
13	0.00020	0.00022	51	0.00556	0.00335	89	0.15271	0.10001
14	0.00023	0.00023	52	0.00609	0.00368	90	0.16659	0.10913
15	0.00027	0.00023	53	0.00667	0.00403	91	0.17991	0.11521
16	0.00031	0.00024	54	0.00727	0.00442	92	0.19390	0.12499
17	0.00037	0.00024	55	0.00789	0.00483	93	0.20874	0.13826
18	0.00043	0.00025	56	0.00847	0.00524	94	0.22451	0.15451
19	0.00047	0.00026	57	0.00898	0.00563	95	0.24126	0.17429
20	0.00049	0.00027	58	0.00939	0.00601	96	0.25715	0.19155
21	0.00049	0.00028	59	0.00971	0.00636	97	0.27419	0.20596
22	0.00049	0.00030	60	0.00999	0.00671	98	0.29249	0.22227
23	0.00049	0.00032	61	0.01024	0.00707	99	0.31215	0.23736
24	0.00050	0.00034	62	0.01046	0.00746	100	0.33331	0.25810
25	0.00052	0.00038	63	0.01071	0.00788	101	0.35163	0.28068
26	0.00055	0.00042	64	0.01104	0.00833	102	0.37132	0.30562
27	0.00060	0.00046	65	0.01146	0.00883	103	0.39250	0.33315
28	0.00065	0.00049	66	0.01199	0.00940	104	0.41527	0.36369
29	0.00070	0.00052	67	0.01260	0.01005	105	0.43973	0.39318
30	0.00075	0.00056	68	0.01329	0.01076	106	0.46602	0.42883
31	0.00081	0.00060	69	0.01405	0.01150	107	0.49429	0.46604
32	0.00087	0.00064	70	0.01485	0.01229	108	0.52467	0.50427
33	0.00093	0.00069	71	0.01574	0.01314	109	0.55733	0.54477
34	0.00099	0.00074	72	0.01670	0.01406	110	0.59244	0.58702

Age	Man	Woman	Age	Man	Woman	Age	Man	Woman
35	0.00107	0.00080	73	0.01777	0.01508	111	1.00000	1.00000
36	0.00116	0.00086	74	0.01895	0.01620			
37	0.00127	0.00093	75	0.02026	0.01743			

The calculation for $A_{25:25:\overline{10}|}$ are separated by two steps:

a. Calculate

$$\sum_{k=1}^{10} v^k \, {}_{k-1|}q_{25:25}$$

$$\sum_{k=1}^{10} \left(\frac{1}{1.0575}\right)^k \left({}_{k-1}p_{25}\ {}_{k-1}p_{25} - {}_{k}p_{25}\ {}_{k}p_{25}\right)$$

$$\left(\frac{1}{1.0575}\right)^1 \left({}_{1-1}p_{25}\ {}_{1-1}p_{25} - {}_{1}p_{25}\ {}_{1}p_{25}\right) + \cdots$$

$$\ldots + \left(\frac{1}{1.0575}\right)^{10} \left({}_{10-1}p_{25}\ {}_{10-1}p_{25} - {}_{10}p_{25}\ {}_{10}p_{25}\right)$$

Based on Table 6 that is the Indonesian mortality table, the survival probability for husband and wife, and discount factor (v) in this calculation is following the Table 7.

TABLE 7
TABLE FOR SURVIVAL PROBABILITY AND DISCOUNT FACTOR

$v = \left(\frac{1}{1+i}\right)^k$	$x = Husband = 25\ years\ old$	$x = Wife = 25\ years\ old$
$\left(\frac{1}{1.0575}\right)^1 = 0.945626$	${}_0p_{25} = 1$	${}_0p_{25} = 1$
$\left(\frac{1}{1.0575}\right)^2 = 0.894209$	${}_1p_{25} = 0.9995$	${}_1p_{25} = 0.9996$
$\left(\frac{1}{1.0575}\right)^3 = 0.845588$	${}_2p_{25} = p_{25} \cdot p_{26} = 0.99898026$	${}_2p_{25} = p_{25} \cdot p_{26} = 0.999280129$
$\left(\frac{1}{1.0575}\right)^4 = 0.799611$	${}_3p_{25} = {}_2p_{25} \cdot p_{27} = 0.998430821$	${}_3p_{25} = {}_2p_{25} \cdot p_{27} = 0.998860432$
$\left(\frac{1}{1.0575}\right)^5 = 0.756133$	${}_4p_{25} = {}_3p_{25} \cdot p_{28} = 0.997831762$	${}_4p_{25} = {}_3p_{25} \cdot p_{28} = 0.998400956$
$\left(\frac{1}{1.0575}\right)^6 = 0.715019$	${}_5p_{25} = {}_4p_{25} \cdot p_{29} = 0.997183172$	${}_5p_{25} = {}_4p_{25} \cdot p_{29} = 0.997911739$
$\left(\frac{1}{1.0575}\right)^7 = 0.676141$	${}_6p_{25} = {}_5p_{25} \cdot p_{30} = 0.996485143$	${}_6p_{25} = {}_5p_{25} \cdot p_{30} = 0.997392825$
$\left(\frac{1}{1.0575}\right)^8 = 0.639377$	${}_7p_{25} = {}_6p_{25} \cdot p_{31} = 0.99573778$	${}_7p_{25} = {}_6p_{25} \cdot p_{31} = 0.996834285$
$\left(\frac{1}{1.0575}\right)^9 = 0.604612$	${}_8p_{25} = {}_7p_{25} \cdot p_{32} = 0.994931232$	${}_8p_{25} = {}_7p_{25} \cdot p_{32} = 0.996236185$
$\left(\frac{1}{1.0575}\right)^{10} = 0.571737$	${}_9p_{25} = {}_8p_{25} \cdot p_{33} = 0.994065642$	${}_9p_{25} = {}_8p_{25} \cdot p_{33} = 0.995598593$
	${}_{10}p_{25} = {}_9p_{25} \cdot p_{34} = 0.993141161$	${}_{10}p_{25} = {}_9p_{25} \cdot p_{34} = 0.99491163$

$$\sum_{k=1}^{10} v^k \, {}_{k-1|}q_{25:25} = \left(\frac{1}{1.0575}\right)^1 \left({}_0p_{25}\ {}_0p_{25} - {}_1p_{25}\ {}_1p_{25}\right) + \cdots$$

$$\ldots + \left(\frac{1}{1.0575}\right)^{10} \left({}_9p_{25}\ {}_9p_{25} - {}_{10}p_{25}\ {}_{10}p_{25}\right)$$

$$= 0.008583534$$

b. Calculate

$$v^{10}\ {}_{10}p_{25:25}$$

$v^{10}\ ({}_{10}p_{25} \cdot {}_{10}p_{25})$
Based on Table 7 then,
$v^{10}\ {}_{10}p_{25:25} = (0.571737) \cdot (0.993141161) \cdot (0.99491163)$
$= 0.564926216$

So, the net single premium of joint-life endowment insurance can be shown,

$$A_{25:25:\overline{10}|} = \sum_{k=1}^{10} v^k\ {}_{k-1|}q_{25:25} + v^{10}\ {}_{10}p_{25:25}$$

$$A_{25:25:\overline{10}|} = 0.008583534 + 0.564926216 = 0.57350975$$

The second step to find premium is find the life annuity. Life annuity that using for the calculation is life annuity joint-life n-year temporary life insurance.

$$\ddot{a}_{xy:\overline{n}|} = \sum_{k=1}^{10} v^k\ {}_kp_{xy}$$

$$\ddot{a}_{25:25:\overline{10}|} = \sum_{k=1}^{10} v^k\ {}_kp_x \cdot {}_kp_y \cdot$$

$$\ddot{a}_{25:25:\overline{10}|} = v^1\ {}_1p_{25} \cdot {}_1p_{25} + \cdots + v^{10}\ {}_{10}p_{25} \cdot {}_{10}p_{25}$$

$$\ddot{a}_{25:25:\overline{10}|} = \left(\frac{1}{1.0575}\right)^1\ {}_1p_{25} \cdot {}_1p_{25} + \cdots + \left(\frac{1}{1.0575}\right)^{10}\ {}_{10}p_{25} \cdot {}_{10}p_{25}$$

Based on Table 7 the life annuity joint-life insurance got the result
$\ddot{a}_{25:25:\overline{10}|} = 7.40863821$
Then the annual joint-life endowment insurance premium can be shown as,

$$P_{xy:\overline{n}|} = \text{Rp}1{,}000{,}000{,}000\ \frac{A_{xy:\overline{n}|}}{\ddot{a}_{xy:\overline{n}|}}$$

$$P_{25:25:\overline{10}|} = \text{Rp}1{,}000{,}000{,}000 \frac{0.57350975}{7.40863821} = \text{Rp}77{,}410{,}954$$

So, Rp77,410,954 is the annual premium for joint-life endowment insurance based on Indonesian Mortality Table 2019 for married couples with Rp1,000,000,000 benefit and 10-year protection.

IV. CONCLUSION

Based on the research, the researcher realizes of the limitations in this research. Some of the limitations are premium payments are made annually during the insurance period, the type of insurance is joint-life endowment insurance, payment of benefits paid at the end of the year of death, used the Indonesian Mortality Table 2019, calculation an interest rate of 5.75%, the insured is a married couple who enter work: there are 25 years old for a man and a woman, and use a 10-year protection period with Rp1,000,000,000 for the benefit. Also, after the calculation about the premium by using the limitations as well, it can be concluded that application of Makeham mortality law to calculate the premium can influenced the result of premium. The Makeham mortality law affects premiums through the Makeham parameters A, B, and c because these three parameters influence the survival probability (p_x) value in Makeham. In this research, the Makeham parameters were determined using the trials and errors method based on the interval for each parameter. By inputting the pair of trials and errors parameter into Makeham's survival probability (p_x) formula, select the p_x result with a small difference (error) between p_xTMI 2019 and $p_x Makeham$ is needed to get the perfect survival probability of Makeham. This trial and error method focuses on each Makeham parameter, so it requires 105 trials for each age.

The smallest selection of errors on Makeham p_x will produce a construction table, namely survival probability table based on Makeham law. On the results of construction using trials and errors method with Makeham's law, the results of survival probability at 0-year are $p_0 = 0.995949359$ for a man and $p_0 = 0.99734000002$ for a woman, which mean the probabilities of a person currently aged 0-year surviving before reaching the age of 1-year are 0.995949359 for a man 0.9973400002 for a woman. With a survival probability, it is easy to determine the annual premium for joint life insurance. Based on the formula, the annual premium requires a net single premium and a life annuity. The value of net single premium joint-life endowment insurance based on Makeham law is 0.575205205, while the life annuity is 7.372839121. The premium of joint-life endowment insurance is the division of the net single premium and the life annuity, so the premium of joint-life endowment insurance is 0.078016785. With a benefit of Rp1,000,000,000 the premium annual value becomes Rp78,016,785. The annual

premium which based on Makeham's survival probability gets a higher premium than the annual premium based on TMI 2019, which only gets an annual premium of IDR 77,410,954. So, if the insureds choose a premium calculation based on Makeham law, the insureds will pay a higher premium than the premium from calculation based on TMI 2019.

Furthermore, from this research, it is hoped that further research will use another method for applying the Makeham law because this research used the trials and errors method which has a long computation, so the other researchers can use the Maximum Likelihood Estimation (MLE) and Weighted Least Squares (WLS). Probably the other method will minimize the time, because it doesn't require a long computation to find the Makeham values and also for further research could use another type of the insurance such as n-year pure endowment, n-year term, and whole life insurance.

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